

Boundary condition guidelines for XBeach simulations

Methodology for the representation of infragravity waves in varying water depth and wave conditions



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Dit rapport is onderdeel van het project Plan Zandige Waterkeringen. Plan Zandige Waterkeringen, gefinancierd door het Ministerie van Infrastructuur en Waterstaat, Rijkswaterstaat, STOWA, Waterschap Scheldestromen, Waterschap Hollandse Delta, Hoogheemraadschap van Delfland, Hoogheemraadschap van Rijnland, Hoogheemraadschap Hollands Noorderkwartier en het Wetterskip Fryslân, voorziet in de vernieuwing van het instrumentarium ten behoeve van het beheren, beoordelen en ontwerpen van zandige waterkeringen. Het project Plan Zandige Waterkeringen maakt deel uit van het programma BOI.

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Summary

Within the BOI (“*Beoordeling en Ontwerp Instrumentarium*”) program, Deltares is developing an approach to assess the safety of the Dutch dune system. Within this approach the process-based model XBeach will be further developed, calibrated and validated for this purpose. The objective of the current study is to investigate how the infragravity-wave boundary condition can be applied efficiently in the application of the Dutch dune safety assessment.

Currently, the theory used to determine the infragravity-wave boundary condition for XBeach is only valid for deep water conditions and known to overestimate infragravity-wave energy for a model boundary located in shallow water. This will result in an overestimation of infragravity-wave height close to the shoreline, and subsequently the overestimation of dune erosion volumes. To avoid infragravity-wave height overestimation, the wave boundary condition for XBeach should therefore be defined in deep water. However, this typically leads to very long numerical grid lengths with a large number of computational cells and correspondingly long simulation times.

This study identifies the required depth at the XBeach model boundary to be at least three times the offshore significant wave height, while also ensuring the wave celerity ratio ($n = c_g/c$) is smaller than 0.9. When respecting the required start depth, errors in resulting infragravity-wave heights and related dune erosion volumes decrease considerably. More precisely, infragravity-wave height errors decrease from over 50%, to errors smaller than 10% in a majority (> 98%) of the cases. Errors in dune erosion volumes decrease from 20-50% to errors smaller than 10% in a majority (85%) of the cases.

This study further shows that there are no available conceptual, empirical, or numerical models in literature to reliably provide XBeach wave boundary conditions for shallower water depths than those described above. This is due to a lack of information on wave groupiness at the model boundary, without which the infragravity wave growth in the model will be overestimated.

For application in the Dutch dune safety assessment we therefore propose a minimum water depth at the model boundary based on the offshore wave height ($H_{m0}/d > 0.3$) and wave celerity ($n < 0.9$). In cases where available cross-shore profile data do not reach the required depth, we propose a time-efficient approach to meet the requirements of the boundary conditions. In this approach the profile is extended with a steep artificial profile down to the required water depth. This approach reduces relative errors in infragravity-wave height from up to 200% to less than 10%.

Samenvatting

Binnen het *BOI-project* ("*Beoordeling en Ontwerp Instrumentarium*") ontwikkelt Deltares een aanpak om de veiligheid van het Nederlandse duinsysteem te beoordelen. Binnen deze aanpak wordt het procesgebaseerde model XBeach hiervoor verder ontwikkeld, gekalibreerd en gevalideerd. Het doel van de huidige studie is om te onderzoeken hoe de lange-golf randvoorwaarde effectief kan worden toegepast voor de beoordeling van het Nederlandse duinsysteem.

Momenteel is de theorie die wordt gebruikt om de gebonden lange-golf randvoorwaarde voor XBeach te bepalen alleen geldig voor grote waterdiepten en is het bekend dat lange-golf energie wordt overschat wanneer een modelrand in te ondiep water wordt opgelegd. Dit zal vervolgens leiden tot een overschatting van lange-golf hoogte dicht bij de kustlijn, en daardoor in een overschatting van duinerosie volumes, en een onderschatting van stormbestendigheid van duinen. Om overschatting van lange-golf hoogte te voorkomen, moet de lange-golf randvoorwaarde voor XBeach daarom in dieper water worden gedefinieerd, maar dit zou leiden tot een groot aantal numerieke rekencellen en bijbehorende lange rekentijden.

In deze studie is de vereiste startdiepte van het XBeach model afgeleid als minimaal drie keer de korte-golf hoogte op diep water, waarbij de golfsnelheidsverhouding ($n = c_g/c$) kleiner moet zijn dan 0.9. Bij het gebruik van de vereiste minimale startdiepte, nemen de relatieve fouten in lange-golf hoogte en duinerosie-volume aanzienlijk af. Zo worden de relatieve fouten in lange-golf hoogten beperkt van oorspronkelijk > 50% tot fouten kleiner dan 10% in een meerderheid van de simulaties (98%). Relatieve fouten in duinerosievolumes nemen af van oorspronkelijk 20-50% tot fouten kleiner dan 10% in een meerderheid (85%) van de gevallen.

In deze studie stellen we vast dat er momenteel in de literatuur geen conceptuele, empirische of numerieke modellen beschikbaar zijn om op betrouwbare wijze XBeach-golfrandvoorwaarden voor ondieper water op te leggen dan hierboven beschreven. Dit is te wijten aan een gebrek aan kennis over golfgroepen aan de modelrand, zonder welke de groei van de lange golf in het model sterk kan worden overschat.

Voor toepassing in de Nederlandse duinveiligheidsbeoordeling stellen we daarom een minimale waterdiepte op de modelgrens voor, vastgesteld op basis van de golfhoogte op diepwater ($H_{m0}/d > 0.3$) en de bijbehorende golfsnelheid ($n < 0.9$). In gevallen waarin de beschikbare kustdwarse profielgegevens niet tot de vereiste diepte reiken, stellen wij een tijdsefficiënte aanpak voor om aan de eisen van de randvoorwaarden te voldoen. Bij deze benadering wordt het profiel verlengd tot de vereiste startwaterdiepte met een steil kunstmatig profiel. Met deze methode worden de relatieve fouten in berekening van de lange-golf hoogte beperkt tot <10%, bij de oorspronkelijke aanpak werden fouten tot >200% vastgesteld.

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1 Introduction

1.1 Background

General

The Dutch dune system is a primary line of defence against coastal inundation and therefore periodic evaluation is required to assure that it fulfils its function. The current assessment of dune safety uses an evaluation method based on the empirical DUROS+ model that was originally developed in the 1980s (Technische Adviescommissie voor de Waterkeringen, 1984; Expertise Netwerk Waterveiligheid, 2007). Currently, limitations in this approach due to underlying assumptions of the empirical model restrict the application of this methodology for large stretches of the Dutch coast (*Deltares, 2015*). Furthermore, recent research (*Deltares/Arcadis, 2019*) has pointed to inaccuracies in DUROS+ for large wave period conditions, thereby reducing the validity of the model for the safety assessment of the Dutch dune coast.

In preparation for the next safety assessment cycle in 2023 and the *Beoordeling en Ontwerp Instrumentarium* (BOI) project, Deltares and Arcadis developed an Action Plan for the Safety Assessment of Sandy Coasts (“Plan van Aanpak Vernieuwd Instrumentarium Zandige Keringen”, in Dutch; *Deltares/Arcadis, 2019a*) commissioned by Rijkswaterstaat. The Action Plan for the Safety Assessment of Sandy Coasts, henceforth termed *Action Plan*, describes a transition from the current transect-based safety assessment methodology to an improved, 2DH area-based assessment using the state-of-the-art process-based model XBeach (*Roelvink et al., 2009*).

To ensure inter-comparability of dune assessment results over multiple assessment cycles, the Action Plan proposes a phased development of the new methodology, with four long-term development phases foreseen. The Action Plan describes a set of tasks to be carried out in development Phase 1 to allow for application of the new methodology in a transect-based approach in the dune safety assessment of 2023. These tasks principally focus on the development and validation of the XBeach model, the development and validation of a probabilistic and semi-probabilistic approach, and a redefinition of the assessment methodology using the new modelling approach.

Current subproject

One of the tasks defined in the Action Plan is to describe how wave boundary conditions for the model can be imposed in a consistent and efficient manner. This report describes the way boundary conditions relating to infragravity waves should be imposed in XBeach to ensure reliable results, while maintaining a calculation-time efficient approach.

To determine the bound infragravity-wave boundary condition for XBeach, the theory of Hasselmann (1962) is applied in which infragravity waves are assumed to be in equilibrium with the wind/swell wave groups. However, this theory is strictly only valid for deep water conditions and infinitely long horizontal beds. Application of this theory is known to overestimate the amount of infragravity-wave energy in shallow water. Placing the boundary condition in too shallow water will therefore lead to an overestimation of infragravity-wave height close to the shoreline, and thereby in an overestimation of dune erosion volumes and an underestimation of dune resilience to storms. However, extending the XBeach model domain to deep water conditions (i.e., water depth of say 50 m) is impractical, computationally expensive, and in some cases impossible due to a lack of available bathymetric data. In the Dutch dune safety assessment, XBeach models for the Holland and Wadden Coast profiles would require a long numerical grid with a large number of computational cells and long simulation times. For certain other sections along the Dutch coast like the Zeeland coast, there is no meaningful relation between the 50 m water-depth contour in the central North Sea and the wave conditions at the coast due to the presence of large sandbanks in front of the

coast. In addition, in the current dune safety assessment methodology the hydraulic boundary conditions are defined at a water depth of 20 m (based on the available nearshore buoy data), and corresponding JarKus profiles. Extrapolating the bathymetric profile and de-shoaling the wave conditions from 20 m water depth to deeper water inherently leads to additional uncertainty in model predictions.

The objective of the current BOI subproject is therefore to investigate if and how the infragravity-wave boundary condition can be adapted to start XBeach simulations in intermediate to shallow water.

1.2 Research questions

The main research question is: *“How should wave boundary conditions be applied in the Dutch dune safety assessment to ensure a robust and accurate description of infragravity waves in a time-efficient manner?”*

To answer the main research question, two sub questions are defined:

1. *At what minimum water depth should the XBeach boundary be placed to avoid overestimation of the infragravity-wave height?*
2. *How could the infragravity-wave boundary condition be modified to allow for applications starting closer to shore?*

1.3 Outline

The general model set-up and test approach will be presented in Chapter 2, together with a brief introduction to the currently-implemented infragravity-wave boundary conditions. The first research question will subsequently be addressed in Phase I (Chapter 3), where a sensitivity analysis will be performed to study the effect of water depth and short-wave characteristics on resulting infragravity-wave heights in the nearshore and the related erosion volumes. The subsequent Phase II (Chapter 4) of the study will examine which existing approaches could be used to adapt the infragravity-wave boundary conditions and negate overestimation when starting in shallow water. The most important findings and limitations will be summarized in Chapter 5, where also recommendations for the next phases of the BOI project (and general XBeach model applications) will be given.

2 Approach

2.1 Overall approach

The general model set-up and test approach will be presented below, together with a brief introduction to the currently-implemented infragravity-wave boundary conditions. The first research question is addressed in Phase I, where a sensitivity analysis is performed to study the effect of water depth and short-wave characteristics on resulting infragravity-wave heights in the nearshore and the related erosion volumes. This will lead to a recommendation for a minimum depth on the offshore model boundary. In the following Phase II, existing approaches will be examined that could be used to optimize the model boundary depth and/or the infragravity-wave boundary conditions to negate overestimation when starting in shallower water than recommended in Phase I and thereby decrease calculation times.

2.2 Model set-up and post-processing

The XBeach version used in these analyses is 1.23.5526 *XBeachX* using the hydrostatic mode called *surfbeat*. Following *Roelvink et al. (2018)*, we apply a wave-groupiness-preserving methodology to solve short-wave energy propagation in the model domain. In a manner similar to *Roelvink et al. (2018)* this is achieved by propagating the directionally-spread wave boundary condition in the predefined mean wave direction, which in this case is shore-normal throughout the model domain (no bathymetry variation in the alongshore direction). Similar to *Deltares (2020)*, the value of the *nufac* parameter, which controls the contribution of roller dissipation to viscosity, was changed from 1.0 (default) to 0.0 to overcome numerical instabilities for very small grid spacings. As the focus of the study is on infragravity-wave boundary conditions, which are not affected by this parameter, and all model settings are similar for all performed simulations, this does not affect the general outcomes and recommendations of this study. Most of the analyses is performed in 1D profile mode, but where calculations are performed in 2D, wall boundary conditions are applied on the lateral boundaries. Wall boundaries were applied in 2D mode as no alongshore wave-driven current is expected (waves come in at a 90-degree angle with the coast), to constrain flow and reduce the potential for instabilities. Since the erosion volumes are compared for different profiles, the *morstart* keyword, representing the start of morphology changes in the domain, is set to 1 hour. In this way the traveling time towards the beach in the longer domains does not affect the amount of dune erosion. Since the forcing of infragravity waves are studied, the *epsi* parameter, controlling the forcing of the mean (tidal) current, is set to 0 to prevent additional artificial long wave generation. When using a tidal signal in the simulation, this parameter sets the ratio of the mean current to the time-varying current through the boundary. Specific model settings, based on the WTI settings (*Deltares, 2015b*), are given in Table 2.1.

Table 2.1: Model set-up adopted during the analysis

Model parameter	setting
<i>Flow boundary condition parameters</i>	
left	wall
right	wall
<i>Flow parameters</i>	
bedfriction	cf
bedfriccoef	0.001
nuhfac	0
<i>Wave parameters</i>	
gamma	0.541
alpha	1.262
gammax	2.364
fw	0
beta	0.138
snells	1
single_dir	0 (1 in 2D mode)
<i>Morphology parameters</i>	
Wetslp	0.260
facSk	0.375
facAs	0.123

The computational grid is discretised by an optimization function related to the peak wave length. In this study 150 points per peak wavelength are applied in the cross-shore direction (more information on the grid size resolution can be found in Appendix A.1). In *Deltares* (2020) it was found that the infragravity wave height is not very sensitive to the along-shore resolution for perpendicular incident waves. Therefore, the dy is set to 20 m in 2D simulations.

2.2.1 Wave height post-processing

In the post-processing, the model predictions distinguish between short waves and infragravity waves. These wave parameters are computed for a simulation period of 2 hours, which corresponds to at least 480 waves for the conditions with the longest wave periods (assuming a T_p/T_m ratio of 1.28, where T_m is the mean wave period). This simulation period is long enough to accurately describe the short- and infragravity-wave characteristics. A hydrodynamic spin-up time of 2 hours is applied before the wave parameters are computed. This long spin-up time is needed for the first reflective waves to arrive back at the boundary in the longer domains. The short-wave height is defined as,

$$H_{m0,HF} = RMS(H)\sqrt{2} \quad (2.1)$$

Where H is the time series of short-wave height varying at the wave group scale.

The infragravity wave height is computed as,

$$H_{m0,LF} = 4STD(z_s) \quad (2.2)$$

Where z_s is the time series of the short-wave-averaged water level.

2.2.2 Dune erosion volume

In simulations where dune erosion is computed, the dune erosion volume is defined as the eroded volume above the storm surge level during the storm. In Figure 2.1 in red the erosive zone above the maximum surge level during storm is depicted, which is typically considered during Dutch dune assessments – and therefore also in this study. It is important to note that the wave run-up is not included in the definition of the maximum water level.

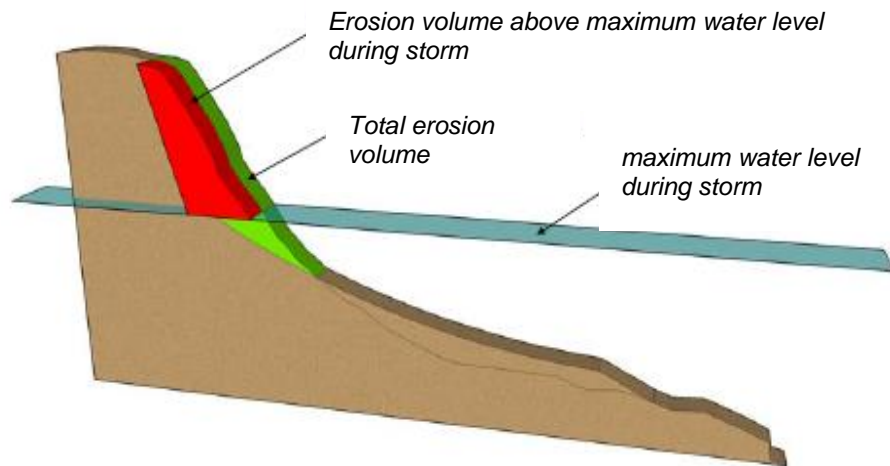


Figure 2.1: Schematic representation of dune erosion volumes. In green the total erosion volume is depicted, and in red the erosive zone above the maximum high-water line during storm, of which the latter is typically considered during Dutch dune assessments – and therefore also in this study.

2.3 Description of XBeach infragravity-wave boundary conditions

2.3.1 Infragravity-wave formation and short-wave groupiness

Infragravity-wave generation is linked to the presence of short-wave groups, which are formed due to the superposition of two different short-wave trains, with wave lengths and frequencies that are very similar. When the waves are in phase their amplitudes are added, and when they are out of phase their amplitudes damp each other out (Figure 2.2a). This results in a wave group structure (Figure 2.2b), which is irregular in shape due to the various frequencies present in a natural wave field. The bound infragravity waves are formed because the larger short waves in the short-wave group transport more momentum than the smaller short waves (variation in radiation stress), leading to a water level lowering under the larger waves, and a relative water level increase under the smaller waves. This induces a variation of the mean water level on a group scale, and results in energy that fluctuates at the same frequency as the wave group. This induced wave is bound to the short-wave group and is 180 degrees out of phase (in exact antiphase).

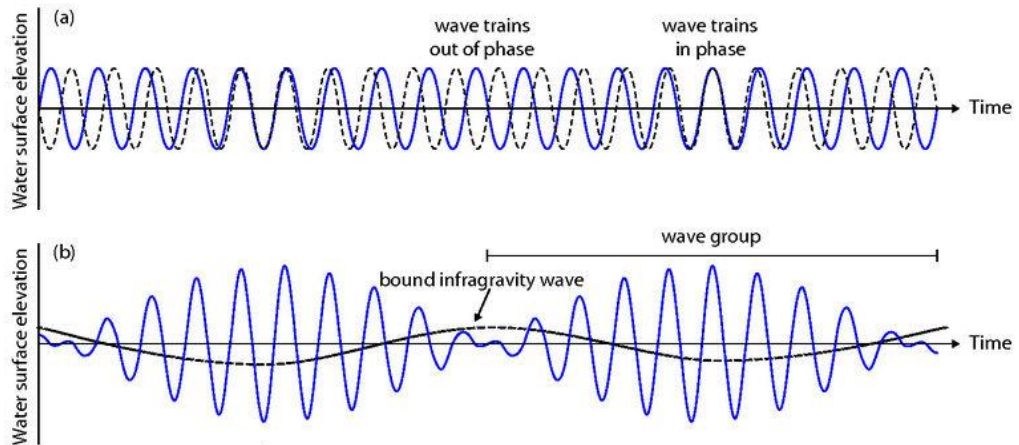


Figure 2.2: (a) The merging of two wave trains of slightly different wave lengths, but the same amplitude. (b) The two wave trains form wave groups and induce a long, bound wave. Modified from Open University (1994)

In shallower water, the groupiness of a signal will change due to shallow water effects (e.g. wave breaking; Figure 2.1). The short wave groupiness represents the number of short-wave groups present in a typical timeseries. Here, the groupiness is computed as,

$$GF = STD(H)/MEAN(H) \quad (2.3)$$

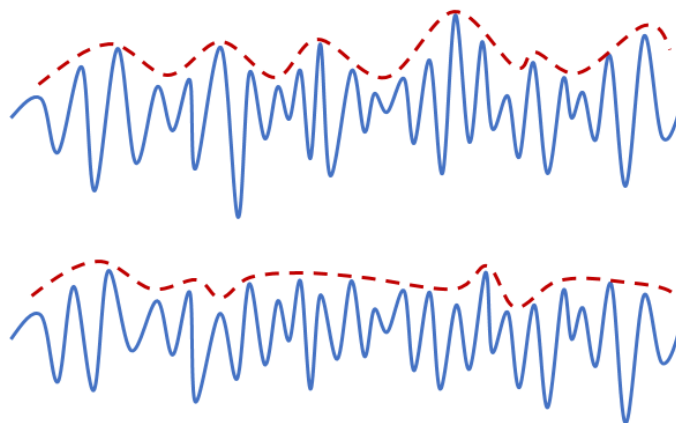


Figure 2.3: Schematic wave group signal in deep water (upper signal) and shallow water (lower signal). The red dashed line represents the wave height, showing greater wave groupiness in the upper signal than the lower signal.

2.3.2 Infragravity-wave boundary conditions in XBeach

The infragravity-wave boundary condition is here obtained by assuming a local equilibrium between the directionally-spread incident sea-swell wave forcing and the bound IG waves. The contribution of the free incident IG waves is thus ignored, due the absence of more detailed information. A recent study of Reniers et al. (2020) has investigated the validity of this approach, and analysed observations of infragravity waves at three measurement stations in the North Sea in water depths of order 30 m. More specifically, they examined the potential contribution of free and bound infragravity waves to the total infragravity wave height. They conclude that, although variable depending on storm characteristics, the ratio between the predicted bound- and observed total IG variance is typically high at the peak of a storm. These findings confirm the validity of the XBeach model approach.

The calculation of the bound infragravity-wave boundary conditions for the XBeach model is detailed here below. In XBeach-Surfbeat mode two different boundary conditions are needed at the boundary to force the model, namely a time-varying wave group signal of the short waves and the corresponding (infragravity) wave signal. Both time series are generated internally by the XBeach model based on a random wave field related to a user-defined input wave spectrum. The wave group signal is given by the envelope of this random wave field. Using the theory of *Hasselmann et al. (1962)* implemented by *Van Dongeren et al. (2003)*, the infragravity water level signal is constructed through the summation of all wave-interactions of a pair of short waves. In 1D mode (normally-incident waves without directional spreading), the solution reduces to the original theory of *Longuet-Higgins and Stewart (1960)*. An interaction coefficient determines the amplitude of the second-order wave for each pair of short waves. By summing up all these second-order waves, the infragravity water level signal is obtained. For each pair of short waves an interaction coefficient determines the amplitude of the second-order wave.

It is important to note that the interaction coefficient, and hence amplitude of the infragravity waves, is related to the water depth, the frequencies of the interacting short-wave components and the wave directions of the interacting wave components. Shallower water depths, lower frequencies and smaller difference angles result in higher amplitudes of the second-order wave. Thus, the equilibrium bound infragravity wave height becomes larger for wave conditions with a lower peak frequency and less directional spreading, and for shallower water depths. Moreover, most of the infragravity wave energy is found by the frequencies close to zero for a wave field without directional spreading (see Figure 2.4). This means that in the 1D simulations most of the energy is present at very low frequencies. When directional spreading is included (2D simulation with directional spreading), the infragravity wave energy decreases and the maximum energy is found at higher frequencies.

This second-order wave theory is only valid for a horizontal bed and a small wave height over depth ratio. This means that the theory assumed a horizontal bed of infinite length seaward of offshore boundary, and that the solution is in equilibrium. However, in reality the bathymetry is not flat, and the infragravity-wave growth during shoreward propagation is slightly lagging behind the equilibrium potential, and they therefore remain smaller. In relative deep water, this will not result in large errors compared to the boundary conditions since the non-linear infragravity-wave growth effects are limited. In shallower water, this will give a larger deviation of the infragravity wave height.

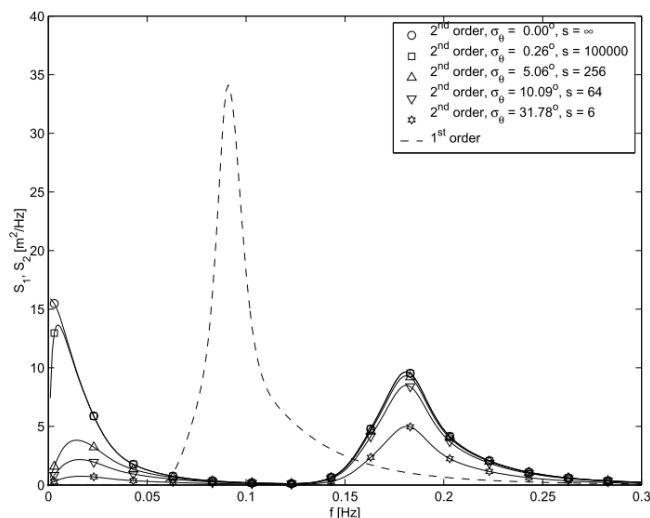


Figure 2.4: Second order wave spectrum given a JONSWAP spectrum for different directional spreading's (Klopman, G., & Dingemans, M. W., 2001).

2.4 Transformation of the boundary conditions

The hydraulic conditions are typically given at the 20 m depth contour. However, the observed cross-shore profiles do not always start the same depth of the hydraulic boundary conditions. When the 20 m depth hydraulic boundary conditions are forced at a different depth in XBeach, this could result in a significant error due to shoaling of the wave energy. Therefore, the wave height needs to be de-shoaled to the starting depth of the XBeach model. This is a valid operation when the waves are not breaking and the bed level changes between the 20 m depth contour and the starting depth only result in a shoaled wave height. With respect to a water depth of 20m, the de-shoaled wave height is given by,

$$H_{m0,shoal} = \sqrt{\frac{c_g(d=20\text{ m})}{c_g(d_{start})}} H_{m0} \quad (2.4)$$

Where $H_{m0,shoal}$ is the de-shoaled wave height, c_g the group velocity at a given depth and H_{m0} the offshore wave height. The group velocity is depended on the local water depth and the wave period. Since XBeach solves the energy balance for the spectral period ($T_{m-1,0}$), the shoaling coefficient needs to be computed with the spectral wave period.

2.5 Selection of hydraulic boundary conditions

The representative storm conditions that will be tested here are based on hydraulic boundary conditions for dune safety assessment of the Holland Coast: a significant offshore short-wave height (H_{m0}) of 9 m, wave period (T_p) of 12 s and a surge level of NAP + 5 m, corresponding to an approximate 10,000-year return period (see e.g., Deltares 2020). In addition, a directional spread of 30 degrees was selected based on a simple analysis of observed directional spread at the Europlatform in the period 2018–2019 for 2DH simulations. Variations on this reference condition are tested, ranging from wave heights of 2 to 9 m, and wave periods of 6 to 19 s, to study the sensitivity of resulting nearshore infragravity-wave height to offshore short-wave period and wave height.

2.6 Cross-shore profile

The reference profile used in this study is a representative profile for the Dutch coast and has a dune top located at NAP +15 m. The slope of the dune face is 1:3 and ends at NAP +3 m. From there on the slope is 1:20 to a level of NAP+0m. From NAP+0m to NAP -3 m the slope is 1:70. From that point on seaward the slope is 1:180, see Figure 2.5. No bars or channels are present on the shoreface. The same profile was used in earlier dune erosion research (e.g. WL | Delft Hydraulics, 1982) as well as in the pre-phase of the BOI project (Deltares 2020).

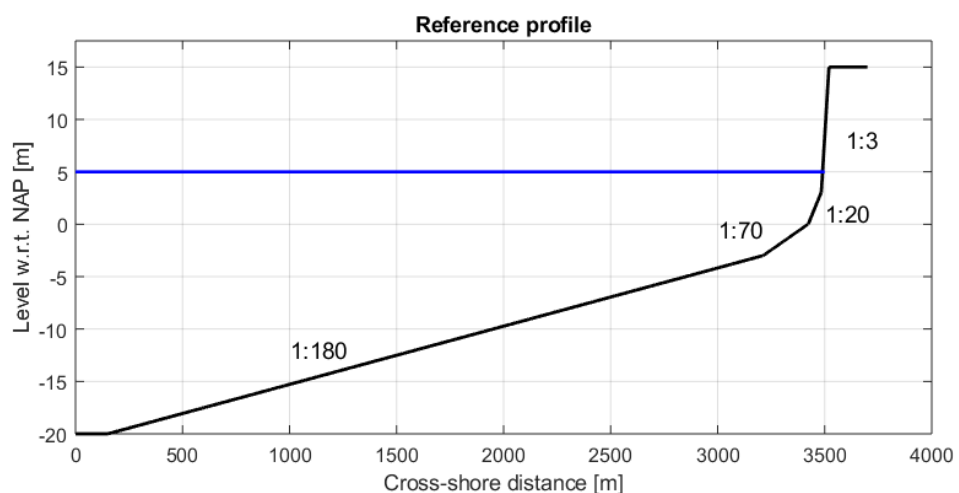


Figure 2.5: Reference profile of the Holland coast based on WL | Delft Hydraulics (1982).

The reference profile has been extended to 50 m depth with a 1/1000 shoreface slope (see Figure 2.6), based on information given in *The Kustgenese 2.0, Atlas of the Dutch Lower Shoreface* (Report 1220339 -ZKS – 0068).

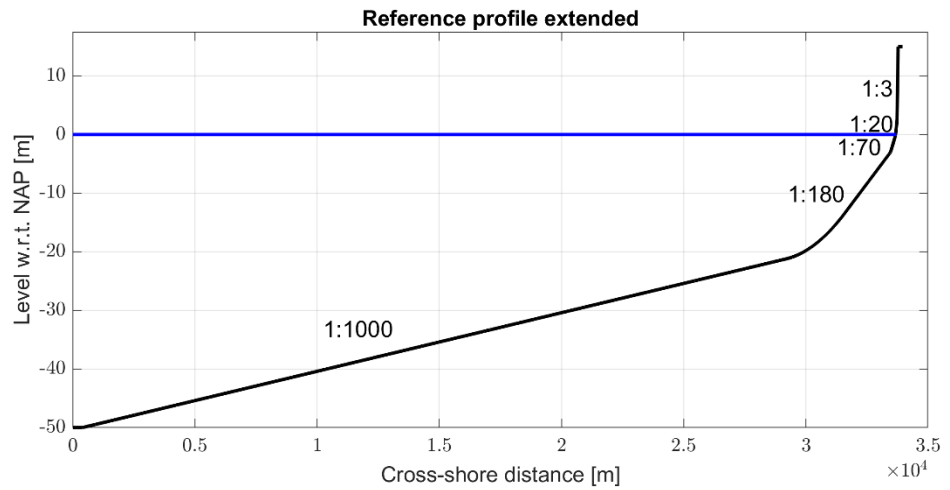


Figure 2.6: Reference profile of the Holland coast extended with a 1:1000 slope to a depth of 50 m.

3 Phase I: Sensitivity analyses on offshore water depth

3.1 Approach

To check at what offshore water depth the infragravity-wave boundary needs to be imposed to accurately force an XBeach-Surfbeat model, the short- and infragravity-wave propagation over a representative cross-shore profile of the Dutch Holland Coast is modelled for numerous short-wave conditions and various boundary depths. The assessed short-wave conditions are defined based on variations of the relative wave height (H_{m0}/d) and the wave celerity ratios ($n=c_g/c$), that together control the infragravity-wave height development. The wave celerity ratio is a measure of the dimensionless water depth (kh) and can be used to identify whether waves propagate in shallow, intermediate or deep water. The theory used to define the offshore bound infragravity-wave height is only valid for small short-wave amplitudes (see Section 2.3). Since the resulting infragravity-wave height is in practice largely related to the local water depth, it could be that a condition with a relatively large short-wave height in deep water does not result in a significant error of the infragravity-wave height. However, the same short-wave height would give a significant error in more shallow water.

Wave conditions were chosen in such a way that the variations in H_{m0}/d ranges from 0.03 to 0.9 and variations in n ranged from 0.55 till 0.96 (see Figure 3.1). This means that wave breaking conditions were also included at the boundary ($H_{m0} > 0.7d$). To obtain the variation in these dimensional parameters, the offshore wave height is varied from 1 to 9 m with a step of 2 m, the peak period is varied from 8 to 20 s with a step of 3 s and the offshore water depth is varied from 10 to 35 m with a step of 5 m. Furthermore, the water level is kept constant at 2 m above the dune foot (corresponding to + 5 m NAP) to simulate dune erosion for storm conditions. The values of the celerity ratio correspond to both shallow water and intermediate water conditions. It is important to note that conditions with $n > 0.96$ have not been tested, but that this would be for cases with very shallow start water depths where a specific approach 'Toets op Maat' is required. The resulting offshore infragravity-wave heights approximated by the theory of *Hasselmann (1962)* range from 0.1 to 1.5 times the incident offshore short-wave height (Figure 3.2). When the start depth is deeper than the start depth of the reference profile (20 m), the domain is extended with a shoreface slope of 1/180 to the required offshore water depth. The offshore wave heights are shoaled or de-shoaled with respect to the wave condition at 20 m. For each wave condition, a reference simulation with an offshore starting depth of 50 m (and a shoreface slope of 1/1.000 up to the 20 m depth contour) is performed. This to be able to compare the equilibrium infragravity-wave height as defined by *Hasselmann (1962)*, and its transformation towards the coast, to the well-developed infragravity-wave height starting from deep water. The examined profiles were here prolonged with a 1/180 slope (not 1/1000 as in the reference case), as resulting differences in wave-height between a 1/1000 and a 1/180 slope were small, and to reduce calculation times. The effect of the shoreface slope will be examined later in this study (paragraph 4.2.2). To show the effects of the prescribed boundary conditions on the predicted dune erosion volumes, the variations in wave conditions were computed with and without bed-updating. All the variations are performed in a 1D simulations. The sensitivity for directional spreading is shown in an additional 2D simulation.

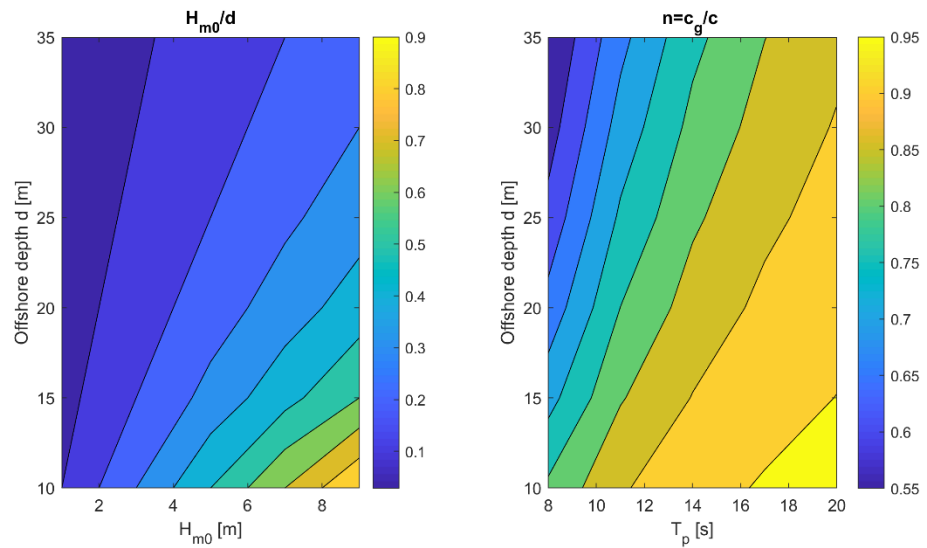


Figure 3.1: Overview of the boundary conditions for the sensitivity analyses on start depth, for the relative wave height (left panel) and the wave celerity ratio (right panel). The left panel shows the relative wave height as function of the wave height and offshore water depth. The right panel shows the relative water depth as function of the peak period and offshore water depth. The offshore water depth is given as the total water depth as the boundary including surge. The offshore wave height is varied from 1 to 9 m with a step of 2 m, the peak period is varied from 8 to 20 s with a step of 3 s and the offshore water depth is varied from 10 to 35 m with a step of 5 m. Furthermore, a surge of 5 m is applied to obtain realistic dune erosion volumes for storm conditions.

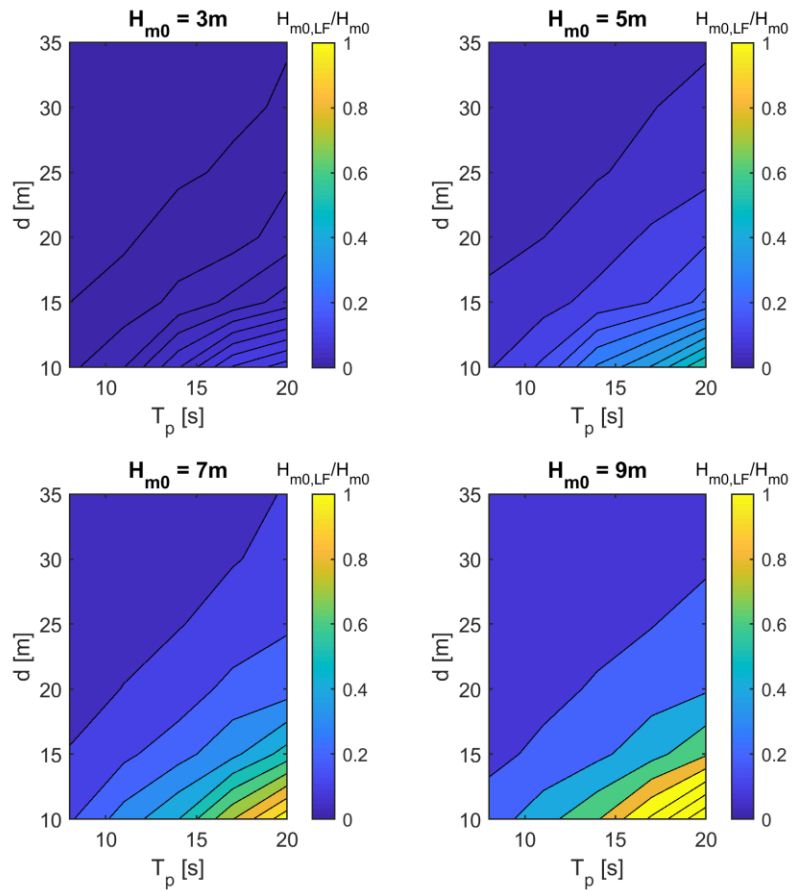


Figure 3.2: Ratio of the equilibrium bound infragravity wave height (Hasselmann, 1962) over the incident short wave height at the boundary for the matrix of wave conditions. In each subplot, the ratio of the infragravity wave height over the incident wave height is plotted as a function of the offshore water depth and peak period. The different panels show the results for different offshore wave heights.

3.2 Sensitivity analyses on offshore water depth

3.2.1 Wave characteristics

When the wave boundary is imposed in too shallow water the infragravity-wave height is highly overpredicted compared to the reference simulation (Figure 3.3). In the case of a wave height of 9 m and a peak period of 14 s, an offshore water depth deeper than 25 m is required to obtain similar results as the reference condition (black line). A smaller water depth at the boundary results in an overestimation of the infragravity-wave height, because the boundary condition for the infragravity waves is no longer valid. Furthermore, the shoaled short-wave height does not match the short-wave height transformation of the reference condition in too shallow water since the offshore depth is located in the breaking zone.

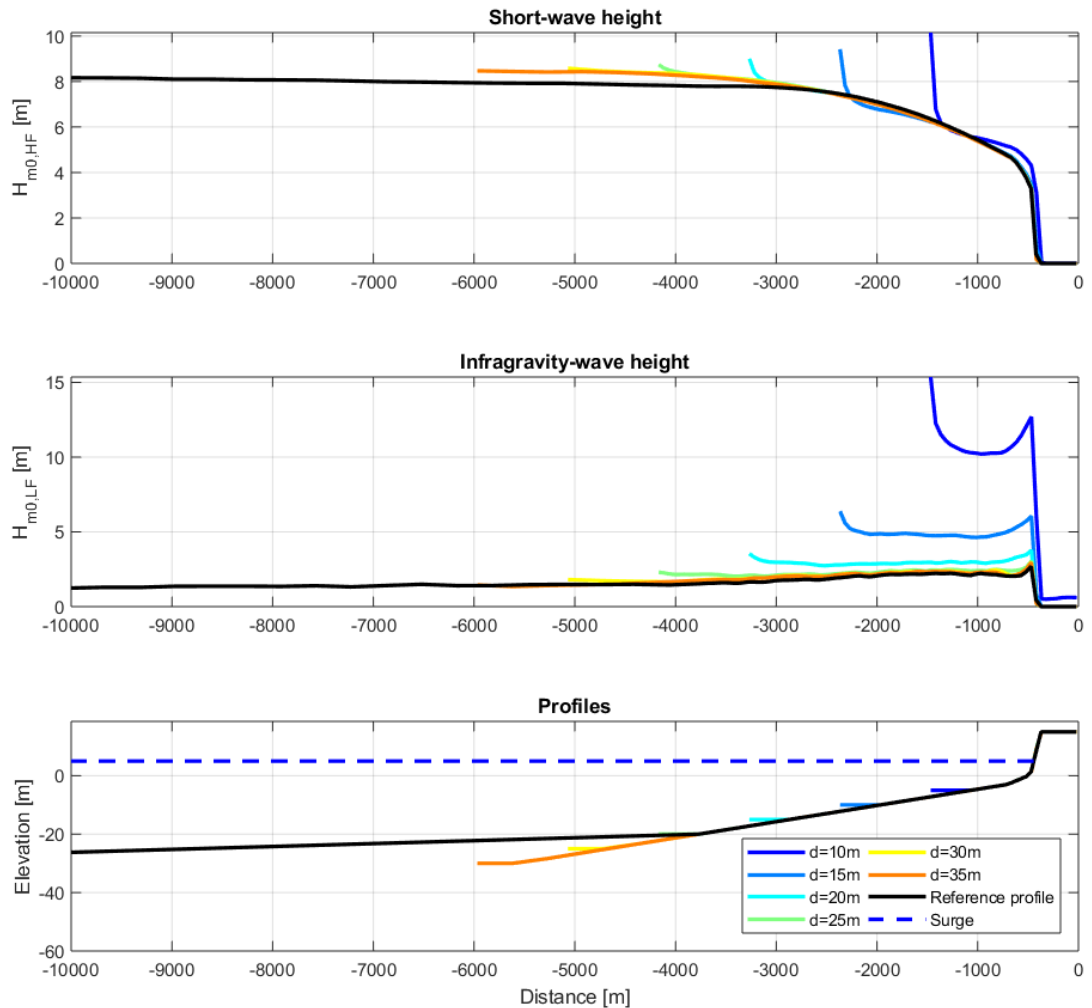


Figure 3.3: Wave transformation of the short-wave height (upper panel), infragravity wave height (second panel) for different offshore depths. The third panel shows the bathymetry of the different profiles. The results of the reference conditions are shown with a black line and the coloured lines show the results of the profiles with a different start depth. These results were obtained by an offshore wave height of 9 m and a peak period of 14 seconds. Note that only part of the reference profile is visible in subplots. The reference profile is a representative profile of the Holland coast extended with a 1:1000 slope to a depth of 50 m (see Figure 2.6 for more information).

For every wave condition, the predicted total infragravity-wave height by XBeach at 5 m water depth (dune toe) is compared to the prediction of the deep-water reference simulation. Results were similar at other locations, although more sensitive to noise in shallower water (not shown). The relative error is normalized by the offshore short-wave height ($(H_{m0,LF} - H_{m0,LF,ref}) / H_{m0}$), thereby considering the significance of the error in predicted infragravity-wave height. For example, some wave conditions with a very low infragravity-wave height ($H_{m0,LF} \approx 0.1$ m) resulted in a relatively large error compared to the deep water reference condition, while this error is not relevant compared to the short-wave conditions at the beach.

The normalized relative errors in infragravity-wave height ($(H_{m0,LF} - H_{m0,LF,ref}) / H_{m0}$), are plotted versus both the dimensionless wave height (H_{m0}/d) and wave-celerity ratio ($n = c_g/c$), to show their dependence on those parameters (Figure 3.4). For relatively small dimensionless wave heights a larger wave-celerity ratio is acceptable and vice-versa, a lower wave celerity is required for larger dimensionless wave heights (Figure 3.4). Most of the relative errors ($\approx 60\%$) are smaller than 10% with a maximum relative error of 240% (Figure 3.5). The transition between the region with

acceptable results (relative error <10%) and larger errors is shown with a dashed red line. For conditions exceeding this dashed line, the nearshore infragravity-wave height could be significantly overestimated. This 10% threshold in wave-height error has been defined after validation with the resulting erosion volumes errors (see next paragraph 3.2.2).

The two thresholds should therefore be used as a criterium to prescribe an infragravity-wave height at the boundary which would result in accurate infragravity-wave heights in the nearshore. This is linked to second-order wave theory, which is only valid for deep water. When the dimensionless short-wave height becomes too large, the boundary conditions result in an overestimation of the infragravity-wave height. Since the resulting infragravity-wave height is larger for longer-period short waves, the error is more pronounced for a larger wave-celerity ratio.

Since respecting the limitations described by the line would result in a rather complex definition of the required start-depth, a more practical rule is advised, namely that: **the XBeach boundary conditions should be located at a water depth where both $n < 0.9$ and $H_{m0}/d < 0.3$** . When respecting these thresholds, 98% of the data has <10% relative error (Figure 3.5).

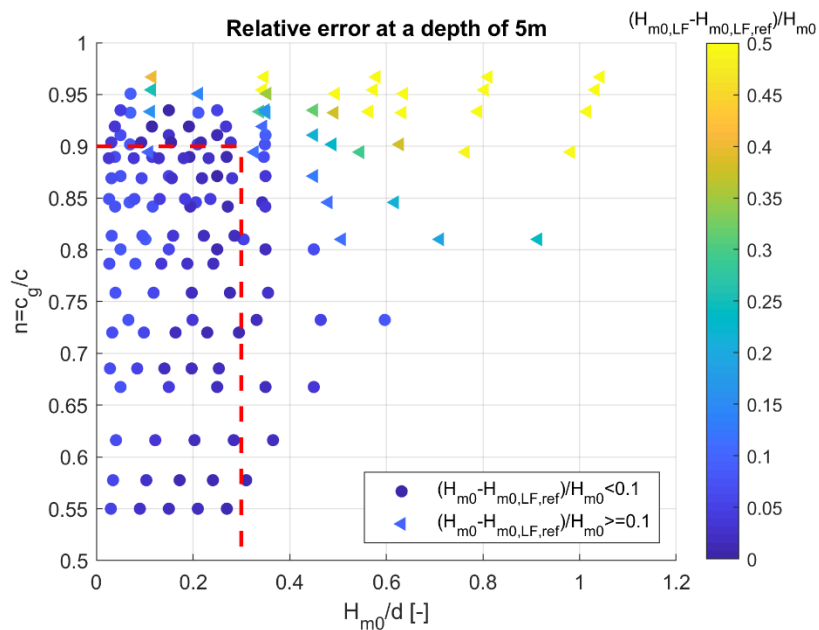


Figure 3.4: Scatter plot of the relative error in the infragravity wave height $((H_{m0,LF} - H_{m0,LF,ref}) / H_{m0})$ at a depth of 5 m. The offshore short-wave height (including shoaling effects) is used in the dimensionless wave height (H_{m0}/d) . The points with a relative error smaller than 10% are shown as circles and the triangles represent points with an error equal or larger than 10%. The transition between the region with acceptable results (relative error <10%) and larger errors is shown with a dashed red line.

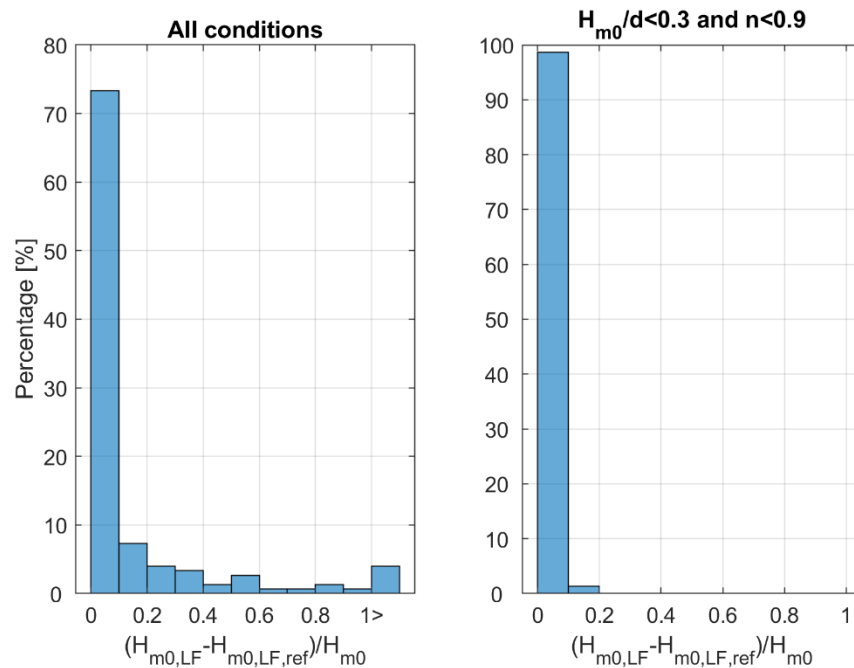


Figure 3.5: Distribution of the relative error in infragravity wave height for all wave conditions (left) and wave conditions respecting the starting depth (right). The blue bars show the percentage of occurrence of the relative error within the corresponding range.

3.2.2 Erosion volumes

In the BOI program, the XBeach model will be applied to assess the Dutch dune resilience. Therefore, the effect of the infragravity-wave height overestimation on the resulting predicted dune erosion volumes is further quantified here. For each wave condition, the dune erosion volume is compared to the dune erosion volume predicted for the reference condition with a start depth at 50 m $((V - V_{ref}) / V_{ref})$. Dune erosion volume errors > 20% are only present for relatively large dimensional wave heights, $(H_{m0}/d > 0.6)$ (Figure 3.6). These relatively large errors are excluded with the imposed thresholds of $n < 0.9$ and $H_{m0}/d < 0.3$ (Figure 3.7). The $n < 0.9$ threshold is not required when only dune erosion volumes are considered, but the threshold is retained since large infragravity-wave height overestimations occur when $n > 0.9$, which is not the case.

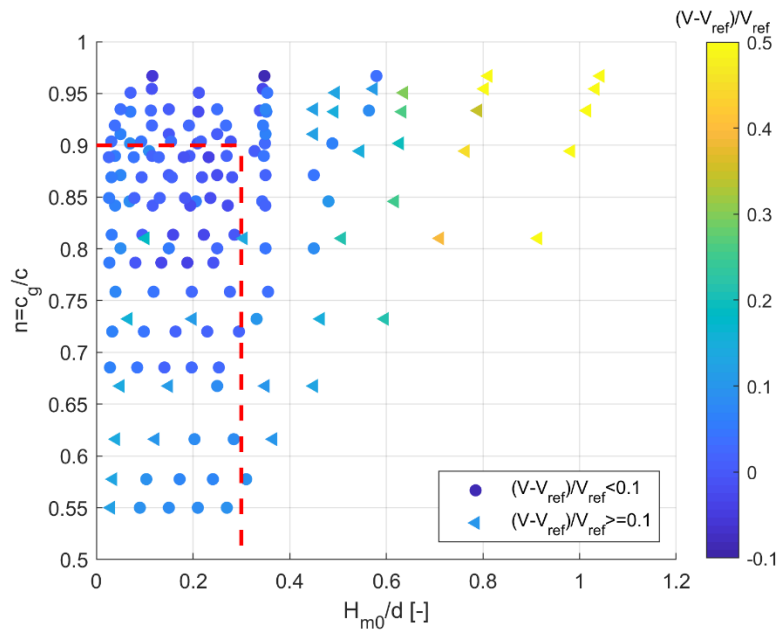


Figure 3.6: Scatter plot of the relative error in dune erosion volumes. The offshore short-wave height (including shoaling effects) is used in the dimensionless wave height (H_{m0}/d). The points with a relative error smaller than 20% are shown as circles and the triangles represent points with an error equal or larger than 20%.

The conditions where $n < 0.9$ and $H_{m0}/d < 0.3$ result in a relative error in dune erosion volume of maximum 18% with the majority ($\approx 85\%$) of the relative errors smaller than 10% (Figure 3.7). Moreover, the largest errors are found for relatively small total dune erosion volumes (Figure 3.7). For these relatively small dune erosion errors, no clear relation can be identified with the magnitude of the infragravity-wave height error. This means that other processes are also affecting the dune erosion volumes. One of the processes responsible for this offset is the random effect of the wave train seeding. To study this, the dune erosion volumes were computed for three different simulations with the same wave conditions, but with different wave trains. These results showed a difference of approximately 2.5% between the various results for a morphodynamical simulation period of 3 hours. This effect partially explains the scatter in the dune erosion volume predictions.

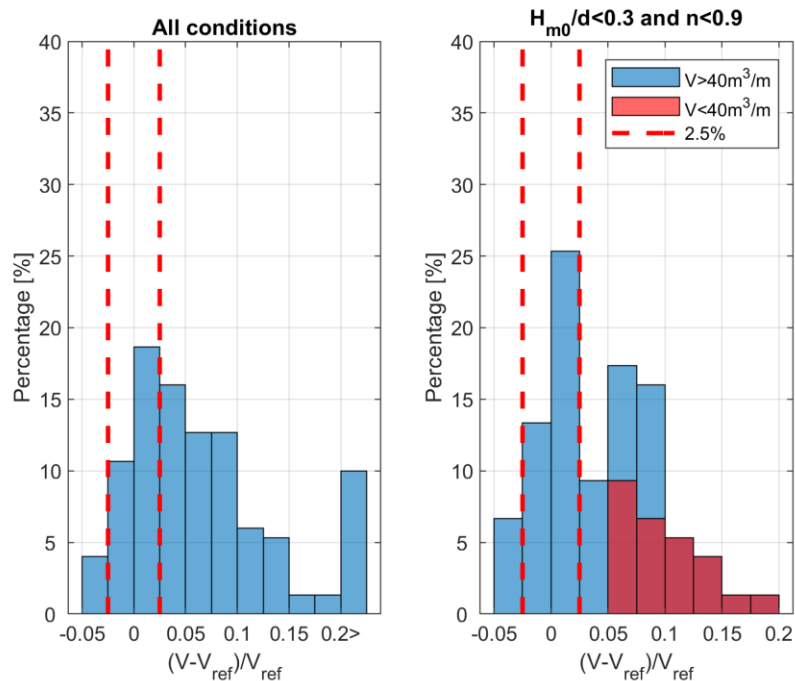


Figure 3.7: Histogram of the relative dune erosion error for all the wave conditions (left) and the condition where the $H_{m0}/d < 0.3$ and $n < 0.9$ (right). The red dashed line shows the 2.5% erosion volume offset, which corresponds roughly with the wave-train seeding effect. In the right panel the colours of the bar show whether the relative error is smaller or larger than $40 \text{ m}^3/\text{m}$.

3.2.3 Effect directional spread

In the above analyses, simulations have been performed in 1D profile mode. Therefore, the effect of directional spread in the short-wave field on the resulting nearshore infragravity-wave height could not be considered. However, in the field some 30° directional spread is expected during storm conditions (based on a simple analysis of observed directional spread at the Europlatform in the period 2018–2019). Generally, infragravity waves are strongly influenced by the directional spread of the short-wave field (e.g., *Okhiro et al., 1992* and *Herbers et al., 1994*). For the case of no directional spread, interactions between the individual waves are strong, leading to strong growth of infragravity waves. Conversely, for larger directional spread as occurs in nature, interactions are weaker and resulting infragravity-wave heights are smaller. This was confirmed in the preliminary phase study of the BOI project (*Deltares, 2020*), where it was identified that inclusion of directional spread in XBeach simulations can lower infragravity-wave heights to at least half their height.

To investigate the effect of directional spread on the error in infragravity-wave height estimation, additional simulations have been performed including 30 degrees directional spread in 2D mode. The domain width is set to 1080 m, corresponding to twice longshore wave group length (L_y) based on the report of *Deltares, 2020*, with $dy = L_y/50$. Results show that the nearshore infragravity-wave heights are considerably smaller when including directional spread in 2D mode, as expected. The variation in infragravity-wave height between the different starting depths is seen to be much smaller when including directional spread (dashed lines in Figure 3.6). This indicates that infragravity-wave height development is less sensitive to the starting depth when directional spread is included, compared to the 1D profile mode. Some 26% overestimation is observed for ($H_{m0} = 9 \text{ m}$, $T_p = 19 \text{ s}$) for a start-depth of 20 m in 1D mode, versus only 8% in 2D mode. For a starting depth of 30 m, with a steeper offshore slope of $1/180$, the offsets decrease for both cases to respectively 8% and 2%,

confirming that the approach of about three times the offshore wave height (here ~27 m) results in minimal overestimation, especially when accounting for directional spread effects.

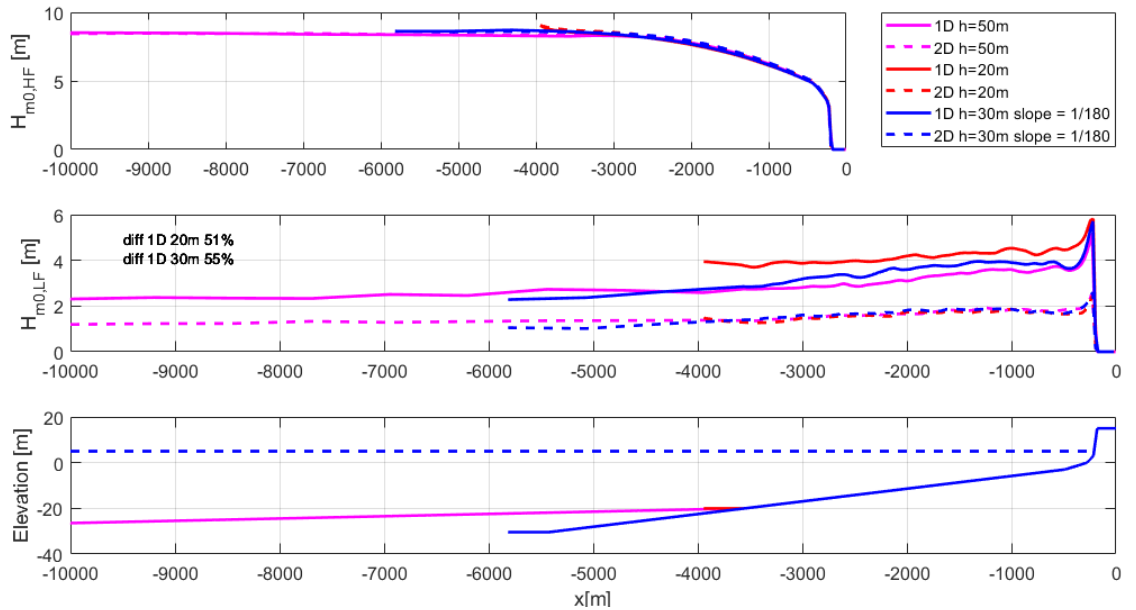


Figure 3.8: Wave transformation over the three cross-shore coastal profiles. (top) Short-wave height transformation, (middle) infragravity-wave height transformation, and (bottom) cross-shore profiles considered. In solid lines the simulations in profile mode, and in dashed lines for 2D mode with 30 degrees directional spread. In pink with a model boundary at 50 m water depth, in red with a model boundary at 20 m water depth and in blue with a model boundary at 30 m water depth, with a 1/180 slope up to 20 m.

When subsequently examining the predicted dune erosion volumes when considering directional spread, the variations between the predictions are even smaller (Figure 3.9). The overestimation of the infragravity-wave height in 1D compared to the 2D simulation (Figure 3.8) is translated in a somewhat larger erosion volume in 1D than in 2D. Overall, the predicted dune erosion profiles are very similar, and while the 1D simulations starting in 20 m water depth without directional spread show some 5% overprediction in dune erosion volume compared to the reference case (start at 50 m depth), these are reduced for the 2D simulations to ~1 % for the simulated conditions. Also, for the simulations starting in 30 m water depth the offsets are relatively small, here underestimating the erosion volumes with 3% in 1D and <1% in 2D mode.

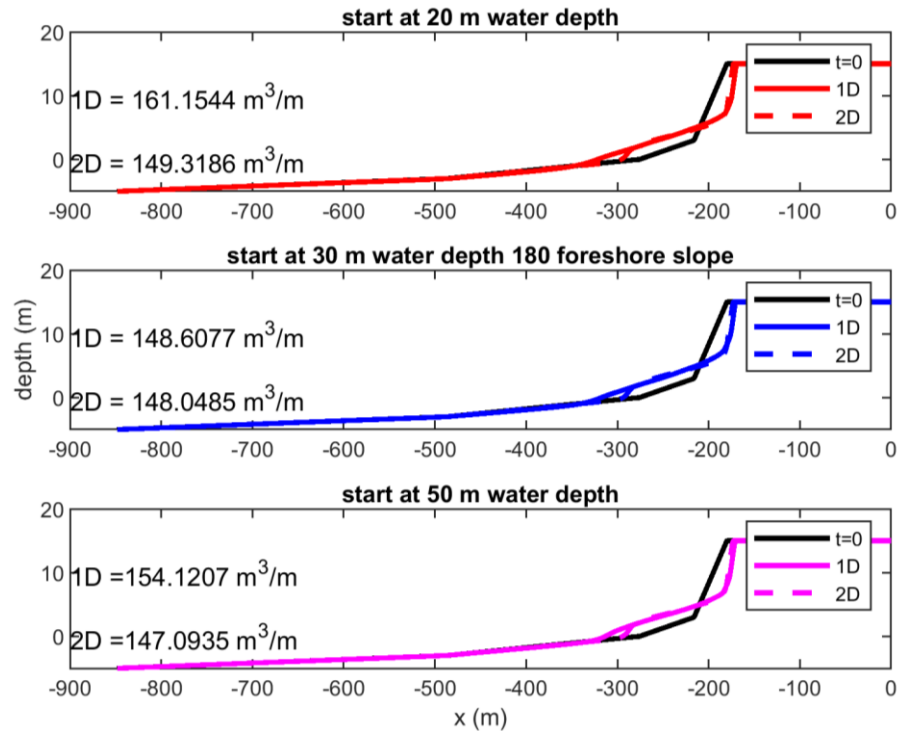


Figure 3.9: Dune erosion during a 4 hrs storm with $H_{m0} = 9$ m and $T_p = 19$ s. (top) Model boundary at 20 m water depth, (middle) model boundary at 30 m water depth, with a 1/180 slope up to 20 m, and (bottom) model boundary at 50 m water depth. In solid lines the simulations in profile mode, and in dashed lines for 2D mode with directional spread.

3.3 Conclusions

Infragravity-wave height predictions by *Hasselmann* (1962) are generally overestimated in too shallow water. To identify the required offshore boundary water depth for XBeach simulations, sensitivity analyses have been performed in 1D profile mode, which show that the required offshore water depth is in general three times the offshore significant short-wave height when respecting $n < 0.9$. For a large storm with $H_{m0} = 9$ m, and $T_p = 18$ s, this would be equal to a start water depth of 27 m. When respecting the condition for the required start depth, resulting dune erosion volumes overestimations are generally $< 10\%$, where the largest errors are present at relatively small dune erosion volumes. Additional simulations including directional spread show that infragravity-wave heights and resulting dune erosion volumes are considerably less overestimated when starting in shallow water, compared to profile-mode calculations.

4 Phase II: Application closer to shore

As observed in Chapter 3 of the current report, XBeach simulations must start in water depths larger than about three times the offshore wave height ($H_{mo}/d < 0.3$) and have a wave celerity ratio smaller than 0.9 to avoid infragravity-wave height overestimation close to shore. This relatively deep starting depth will lead to long cross-shore grids, a large number of grid cells and hence large calculation times. This will create limitations for the practical implementation of the XBeach model assessment.

In addition, the requirement for a starting water depth of three times the offshore wave height is not always met with the available cross-shore profile JarKus measurements along the Dutch coast. For example, in Zeeland the JarKus profiles start in relatively shallow water and start for some locations at a depth of around 5 m below mean sea level. With a representative local offshore wave height of approximately 3.25 m, a peak period of approximately 12 s and a surge of 5 m, the required XBeach start depth limit of ~12 m cannot be respected. Also, at the Wadden coast in the northern part of the Netherlands available JarKus profile measurements regularly do not extend up to the 20 m depth contour, where the required water depth could be up to 30 m.

Two adaptive approaches are considered here in Phase II to reduce the XBeach model domain length and limit calculation times while ensuring robust model predictions. The first approach consists of the application of a reduction factor to the internally-generated infragravity-wave conditions at the boundary (based on the equilibrium bound wave theory of *Hasselmann (1962)* and described in Section 2.3). A second approach that is investigated is the extension of the model domain from the measured data to the required start depth with an artificial slope considerably steeper than the estimated natural slope to reduce the required number of grid cells.

4.1 Inventory of the adaptation approaches

4.1.1 Infragravity-wave height reduction factor

This approach consists of the application of a reduction factor to the internally-generated infragravity-wave conditions at the boundary (based on the equilibrium bound wave theory of *Hasselmann (1962)* and described in Section 2.3). Several methods could be used to determine the appropriate reduction factor and are introduced here below.

- *Look-up table reduction factor*

The most straight-forward approach is the creation of a look-up table for a typical reduction factor of the infragravity wave height based on comparisons with XBeach simulations starting at 50 m (or greater) water depth. This would entail numerous simulations spanning a large number of beach profiles and offshore wave conditions (directional spread, surge, short-wave height and short-wave period).

- *nmax - Reduction factor currently implemented in XBeach*

The *nmax* parameter is presently implemented in XBeach to avoid excessively large infragravity-wave boundary conditions when starting in shallow water. Using the 1D simplification of *Hasselmann (1962)* by *Longuet-Higgins et al. (1962)*, it can be shown that the equilibrium bound infragravity wave amplitude is a function of the difference between the wave group celerity (c_g) and the free long wave velocity (\sqrt{gh} , which equals the short-wave phase velocity in shallow water):

$$\eta = -\frac{S_{xx}}{\rho((\sqrt{gh})^2 - c_g^2)} \quad (4.1)$$

where η is the infragravity wave amplitude, S_{xx} is the cross-shore wave-induced radiation stress, ρ is the density of the fluid, g is gravitational acceleration and h is water depth.

The solution tends to infinity when the short-wave group speed is equal to the shallow-water velocity. The $nmax$ parameter in XBeach limits the group speed to $nmax * \sqrt{gh}$, thereby limiting infragravity-wave heights in shallow water depths. An indication of the appropriate $nmax$ -factor for each application has not previously been well determined.

- *Reduction factor from Zhang et al. (2020)*

A third option for the definition of an appropriate reduction factor is the recently published reduction factor approach of Zhang et al. (2020). They determined a correction factor for offshore infragravity-wave estimations using a large number of simulations for bichromatic 1D wave conditions, water depths ($d = 10, 15$ and 20 m) and slopes ($1/20, 1/40, 1/60, 1/80, 1/100, 1/150, 1/200$). They multiply the bound infragravity-wave energy calculated based on the equilibrium solution of Longuet-Higgins and Stewart (1960) with a local correction factor:

$$\zeta = \zeta_{LHS60} \cdot \left(\frac{h}{h_s}\right)^{\alpha_c} \quad (4.2)$$

The shoaling rate is seen to depend largely on the normalized bed slope β_s parameter, and on short-wave steepness:

$$\alpha_c = \max\left(-10 \cdot \left(\frac{H}{L}\right) + 2.55 - \beta_s^{-0.155}, 0\right) \quad (4.33)$$

Where h is the water depth, h_s is the start of the sloping bed, and the shoaling zone (zone II) in the figure below. H is the short-wave height and L is the short-wave length, together (H/L) forming short-wave steepness. The normalized bed slope β_s is defined following Battjes et al. (2004) as:

$$\beta_s = \frac{h_x}{\omega} \sqrt{\frac{g}{h_s}} \quad (4.4)$$

With h_x as bed slope, ω as angular frequency ($2\pi/T$) and g gravitational acceleration. Their correction-factor approach is seen to correspond well to the bichromatic 1D Van Noorloos laboratory experiments (Van Noorloos, 2003). The predicted bound infragravity amplitude falls within the lower-bound of Green's law for non-breaking free surface gravity waves ($H_1 h_1^{1/4} = H_2 h_2^{1/4}$) and the upper-bound Longuet-Higgins and Stewart (1960) approximation.

An important assumption is the 1D bichromatic wave field – and the relatively small wave heights they considered. To apply the correction factor for a random, quasi-2D approach a new correction factor might need to be determined. This would entail numerous XBeach simulations spanning a large number of beach profiles and offshore wave conditions (directional spread, surge, short-wave height and short-wave period).

- *Linear model approach Reniers et al. (2002)*

A fourth approach could be the determination of a reduction factor using another expression of infragravity-wave amplitude/energy density, as derived by Reniers et al. (2002). They developed a linear model where infragravity waves are forced by directionally spread short waves incident on an alongshore uniform beach, including a friction term. They derive the bound wave energy consistent with the radiation stress approach in their linear model. Integrating the computed infragravity spectra over all directional contributions results in the frequency distribution of the surface elevation infragravity spectra.

4.1.2 Extension of the model domain with an artificial slope

In a second approach, the model domain may be extended with an artificial slope to the required offshore water depth. Preferably, this artificial slope is as steep as possible since a steeper slope will result in less grid cells and hence less computational time. A consequence of this artificial slope is that the transformation (shoaling and breaking) of the waves while they propagate to shore is accelerated, which might introduce inaccuracies.

4.2 Selection of approach

4.2.1 Reduction factor

To test the application of a reduction factor on the infragravity wave-height at the offshore boundary, the n_{max} parameter is applied which can be considered qualitatively (if not quantitatively) representative for all reduction factor approaches described in Section 4.1.1. The four methods described there would only differ in the way to determine the appropriate reduction factor, but the application of the reduction factor in itself is similar.

Since the desired n_{max} value is not known beforehand, a range of n_{max} values is applied. This effect is studied with the reference profile. An offshore wave height of 9 meter with a peak period of 12 seconds, and no surge is applied as forcing at the boundary. The results with a reduction factor at an offshore depth of 20 m are compared to a simulation with an offshore depth of 50 m and a 1/1000 slope until the 20 m water depth.

The results demonstrate that the infragravity-wave height within the model domain cannot simply be adjusted by the application of an infragravity-wave height reduction factor at the boundary (Figure 4.1). Although the infragravity wave height imposed at the offshore boundary could be changed by a reduction factor, the infragravity waves in the domain adapt relatively quickly to a different equilibrium wave height with the short-wave group forcing (see also Section 2.1) than that imposed at the boundary. Due to individual wave breaking of the larger short waves, the groupiness factor (GF) of the short-wave groups decreases in the cross-shore direction (see third panel of Figure 4.1, and a more detailed example in Figure 2.3). As the short-wave transformation in the long domain (start at 50 m depth) is different (wave shape and GF) from the short-wave transformation in the shorter domain (start at 20 m depth), the resulting infragravity wave is different as well. More precisely, the GF of the short waves decreases from a cross-shore position of approximately -20.000 m in the long domain (third panel in Figure 4.1). At a depth of 20 m (-3500 m) in the long domain, the GF is reduced to 0.4, whereas the boundary condition of the short domain starts with a GF of 0.5.

The above example with the n_{max} -approach shows that the reduction factor for the infragravity-wave height at the boundary alone is not a robust solution for the overestimation of the infragravity-wave height in the nearshore. The reduction on the infragravity-wave height is properly imposed at the boundary, but in the domain itself the infragravity wave increases abnormally in height because the short-wave forcing is not yet in balance with the local water depth, and thus too large wave-group forcing induces a too large infragravity-wave. This is independent of the amount of reduction applied on the infragravity-wave boundary. Thus, to correctly model the infragravity-wave behaviour with a boundary defined in shallow water, the long wave behaviour cannot be adapted single-handedly, but also additional information about the short-wave characteristics is needed. None of the methods described in Section 4.1.1 can provide this information.

In the future, an altogether different approach could perhaps be implemented for the definition of the short- and long-wave boundary conditions. The linear model of *Reniers et al. (2002)* is showing potential, as it can help to efficiently propagate the short-waves in to shallower water, and thereby change the local forcing in the model. However, presently the model is not yet providing the required detailed information on short-wave groups needed to force the infragravity wave.

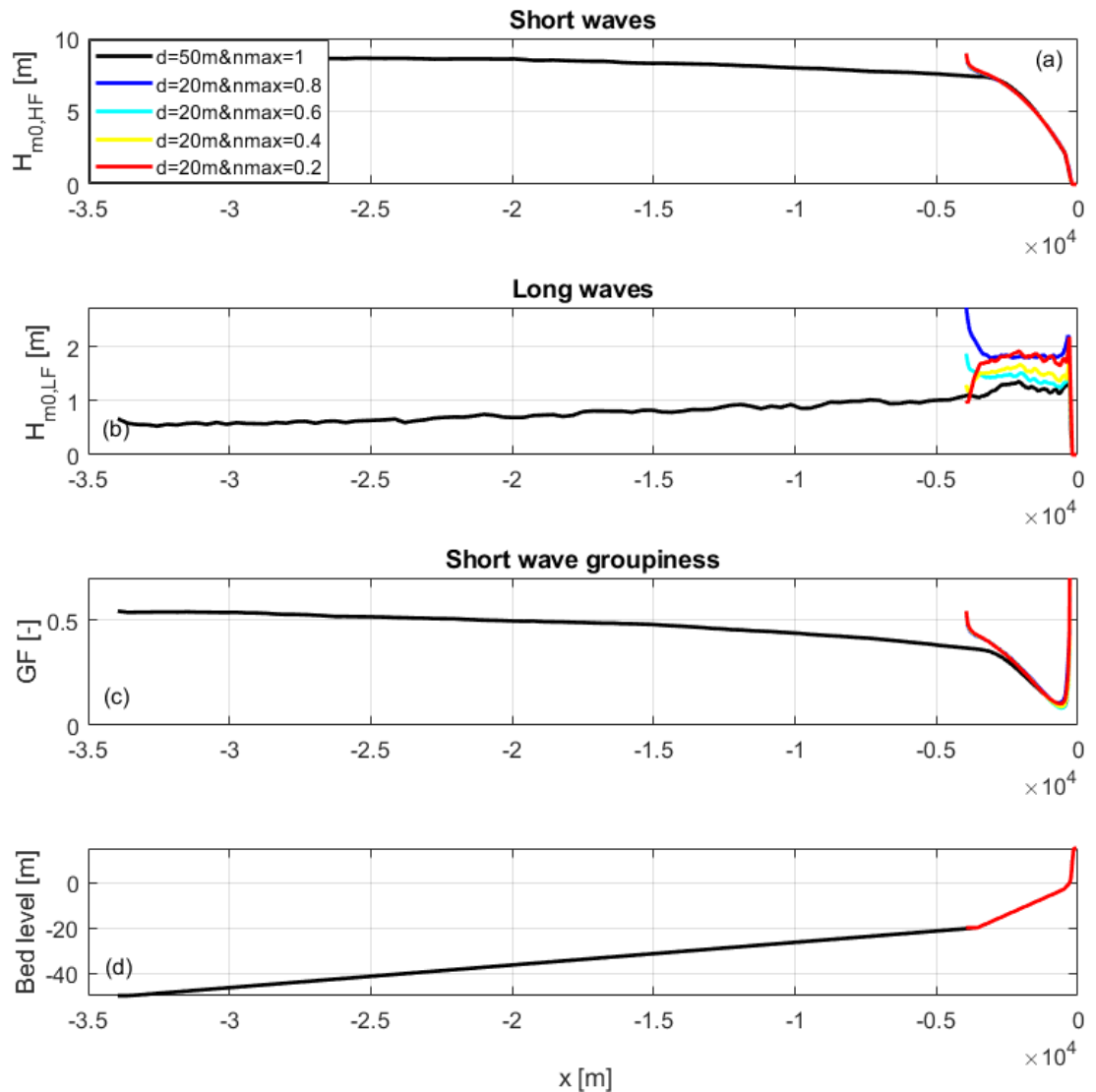


Figure 4.1: Effect of a n_{max} reduction factor on the nearshore infragravity-wave height. Wave transformation over the three cross-shore coastal profiles. (a) Short-wave height transformation, (b) infragravity-wave height transformation, (c) short-wave groupiness, and (d) cross-shore profiles considered.

4.2.2 Steep artificial shoreface slope

An alternative for the reduction factor could be to extend the model domain with a steep artificial slope to the required depth at the boundary. A first test is performed here. As the growth of the infragravity-wave height is known to depend on the beach slope and profile shape, the infragravity-wave height transformation is compared for different lengths of an artificial slope of 1:10 with a reference profile up to the offshore start-depth (Figure 4.2 – bottom subplot). The Holland coast profile that is extended to a bed level of 30 m with a slope of 1:180 m is applied as reference condition, which is deep enough for all tested wave conditions (see Phase I of the current report). The effect of an artificial slope is studied for the most extreme case, with a large wave height of 9 m, a peak period of 20 seconds and a surge of 5 m.

A first test shows that a 1:10 shoreface slope could be used to reduce grid lengths with limited infragravity-wave height error at the shoreline, when starting this slope in not too shallow water (red and yellow lines in Figure 4.2). A starting depth of the steep profile in too shallow water – so that a relatively long part of the profile is steep – results in an underestimation of the infragravity-wave height (blue lines in Figure 4.2). On the steep profile the short waves reach shallow water sooner and have a relatively narrow surf zone compared to the milder sloping profiles. This results in a

limited amount of time for energy exchange from short waves to the infragravity waves on a steep profile compared to a gentle profile (e.g., *De Bakker et al., 2015*). Furthermore, the energy exchange is more pronounced in shallow water when dispersion is small (the short and long waves propagate with a more similar speed) and triad interactions are closer to resonance (*Freilich and Guza, 1984; Herbers et al., 1995*). This makes the infragravity-wave growth more dependent on the applied bathymetry in shallow water.

The application of a steep artificial slope to force an XBeach model at the required start-depth seems promising but demonstrates its limitations. The next section investigates how this artificial slope must be applied to obtain robust predictions of nearshore infragravity-wave heights.

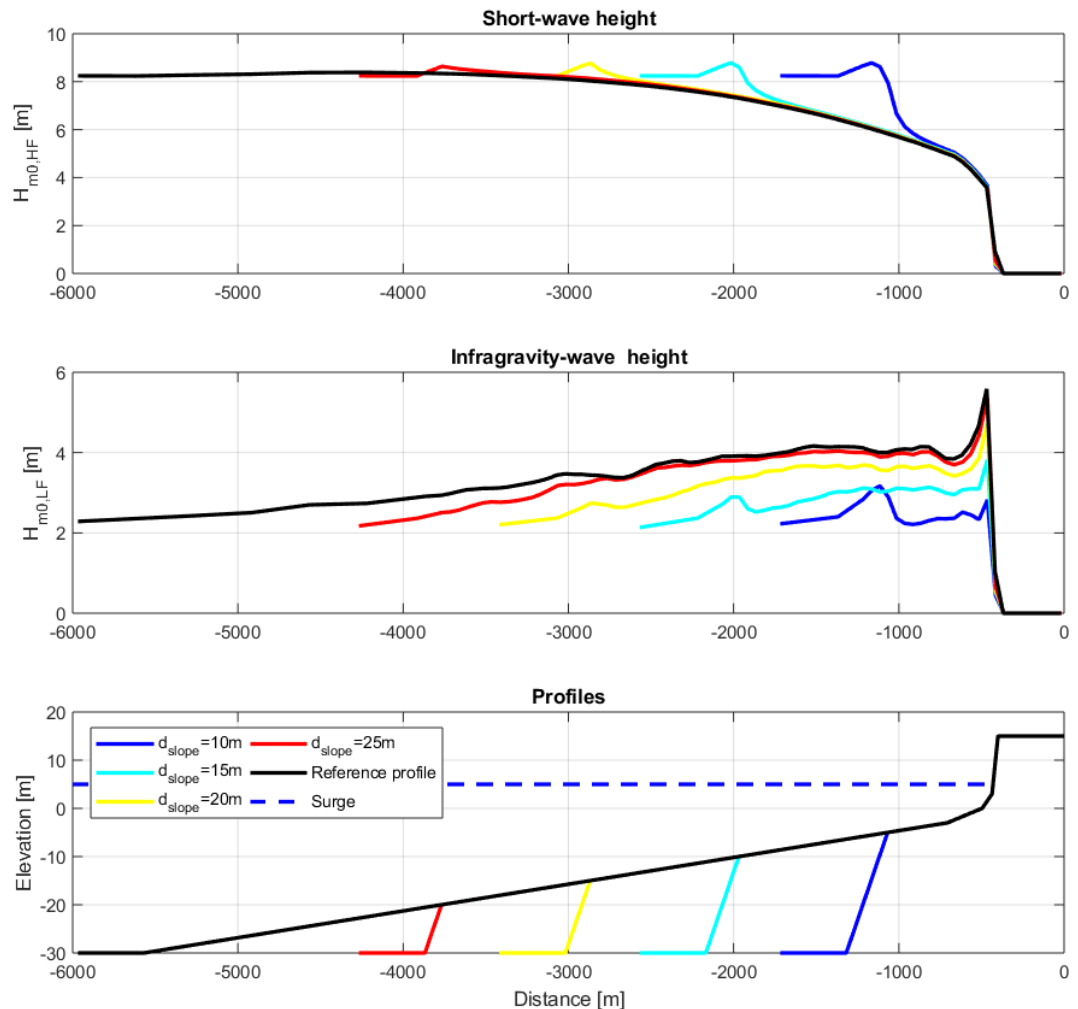


Figure 4.2: Wave transformation over different shoreface profiles compared to the reference condition (black line). The hydrodynamic conditions are compared for the short-wave height (upper panel) and infragravity wave height (second panel). The location of the output-points for the different domains is shown in the third panel. The different colours indicate the results of the various shoreface profiles. The dashed blue line in the lower panel shows the surge level.

4.3 Sensitivity analyses on selected adaptation approach

For applications where the required offshore water depth (as defined in Phase I) is not met, a promising approach seems to be to extend the bed profile with a steep artificial shoreface slope to the required offshore starting depth. This steep artificial slope would limit the need of numerous additional grid cells and therefore long calculation times compared to more gentle artificial slopes.

Here, the sensitivity of the resulting nearshore infragravity-wave height to the shallow starting depth of the artificial shoreface slope is tested for the Holland Coast profile for different wave conditions. Variations in H_{m0} (3, 5, 7 and 9 m), T_p (8, 11, 14, 17 and 20 s) and the starting water depth (including surge) of the shoreface slope (10, 15, 20 and 25 m) are applied. It is important to note that conditions with $n > 0.96$ have not been tested, but that this would be for cases with very shallow start water depths where a specific approach '*Toets op Maat*' is required. The reference profile is defined as the Holland Coast profile extended to a depth of 30 m with a 1:180 slope (offshore water depth of 35 m including surge). All the wave conditions give accurate results at this water depth (see Phase I), which means that the relative error in infragravity-wave height can be expressed in terms of the reference conditions. A 1:10 slope is applied as artificial slope to extend the profiles to a depth of 30 m. The effect of the starting depth and foreshore slope on the resulting nearshore infragravity-wave heights is investigated by comparing the different results in a local water depth of 5 m.

The resulting infragravity-wave heights show a strong dependency on the shallow start-depth of the artificial shoreface slope, with differences in nearshore wave height up to 30%. Differences in simulated nearshore infragravity-wave height increase for shallower start-depths of the artificial slope, and the magnitude of the differences is strongly dependent on the wave period (Figure 4.3). The wave conditions with a peak period of 8, 11 and 14 seconds show a good result for all the starting depths (relative error smaller than 10%). Wave conditions with a peak period of 17 and 20 seconds show relative errors up to 30% when the artificial slope starts in too shallow water.

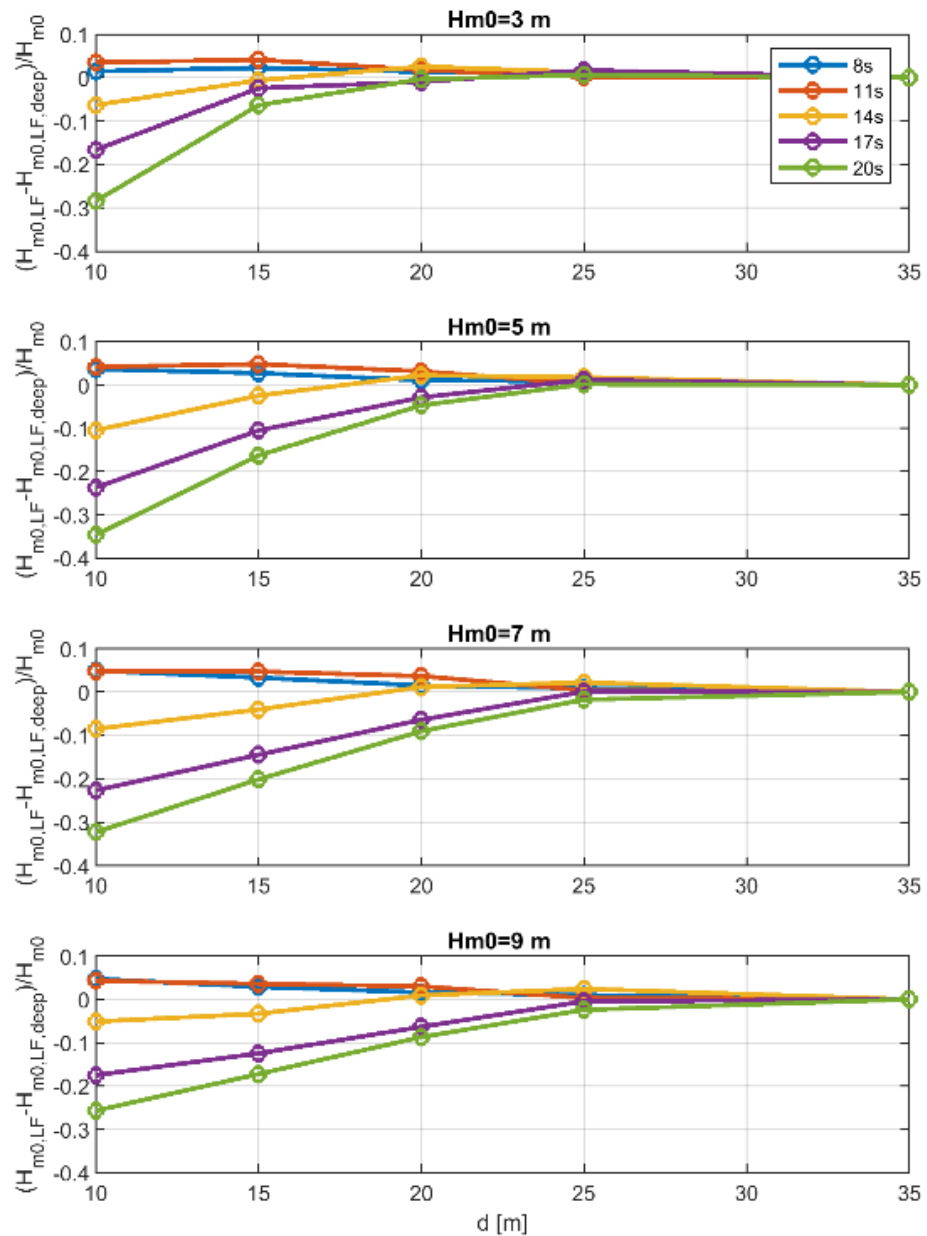


Figure 4.3: Relative error in infragravity wave height as a function of the starting depth of the artificial shoreface slope of 1:10. The different subplots below each other show the results for the variations in offshore wave height. The colours indicate the wave period.

Since the relative error is larger for longer wave periods, the application of an artificial slope of 1:50 was investigated for these conditions (Figure 4.4). This gentler artificial slope reduces the relative error in infragravity wave height to a maximum of 20% with the majority of the relative errors smaller than 10% (Figure 4.7).

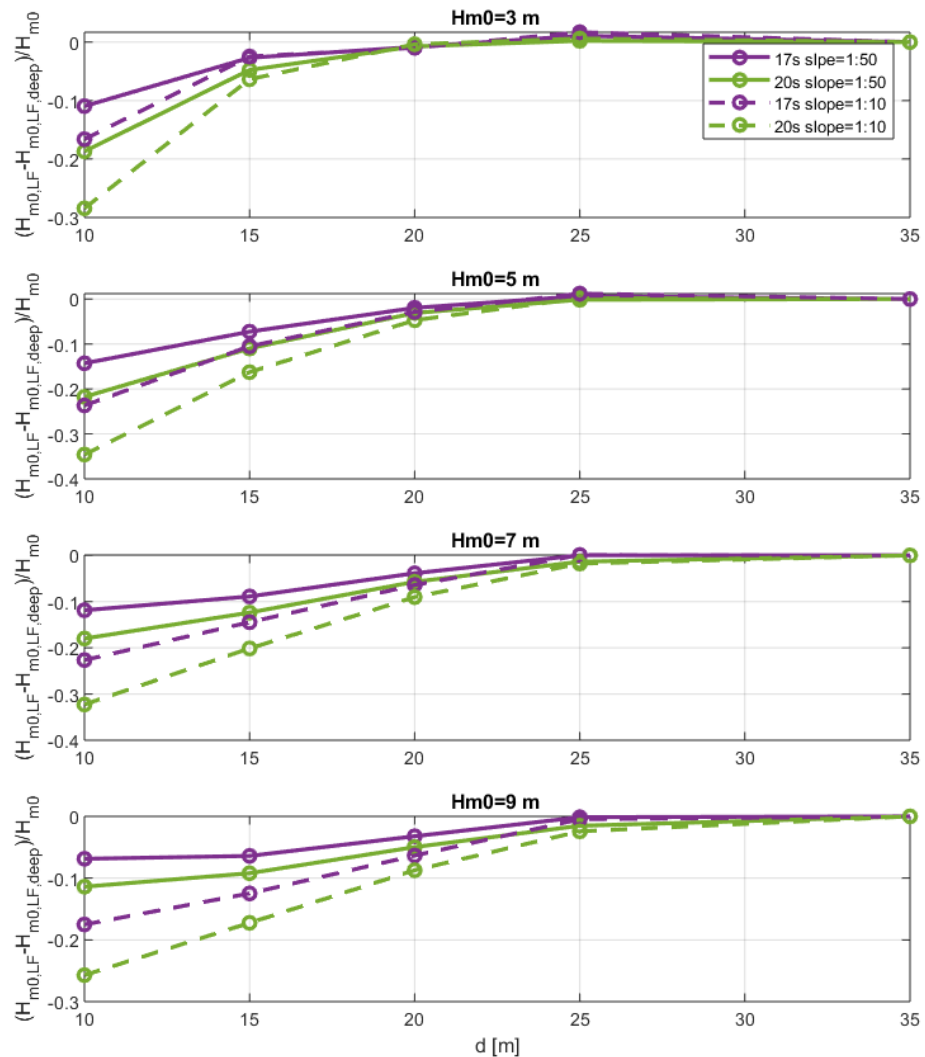


Figure 4.4: Relative error in infragravity wave height as a function of the starting depth of the artificial shoreface slope, for an artificial slope of 1:20 and 1:50. The different subplots below each other show the results for the variations in offshore wave height. The colours indicate the wave period.

To identify the wave conditions and starting depths for which the infragravity-wave height is accurately modelled, the relative error is plotted as a function of the wave celerity and dimensional wave height – similar to Phase I (Figure 4.5 and Figure 4.6). The relative error is seen to be particularly large for large wave celerity ratio's n . This is due to the fact that this ratio is related to the short-wave period, where for large short-wave periods large offsets in the predicted infragravity-wave heights were observed in the figures above.

An artificial slope of 1:10 can be safely applied when the celerity ratio at the upper starting depth of the artificial slope (d_{slope}) is smaller than 0.9 and when $H_{m0}/d < 0.3$ (Figure 4.5, corresponding to Region I of Figure 4.7). This is also the case for Regions II and III (Figure 4.7). This steep artificial slope would limit the need of numerous additional grid cells and therefore long calculation times compared to more gentle artificial slopes. However, when the celerity ratio at the upper starting depth of the artificial slope is larger than 0.9 and $H_{m0}/d > 0.3$ (Region IV of Figure 4.7), a more gentle artificial slope of 1:50 is required to reduce errors significantly (Figure 4.6).

When these thresholds are respected the relative error is smaller than 10% for most of the conditions (100% when $n < 0.9$ or $H_{m0}/d < 0.3$, see Figure 4.7). Only for a few conditions with a long offshore wave period is the relative error larger than 10% ($\approx 30\%$ of the conditions with a T_p of 17 and 20 seconds) with a maximum relative error of 21% (Figure 4.7). Moreover, the proposed method based on an artificial slope shows a significant reduction of the relative error in the region where $H_{m0}/d > 0.3$ and $n > 0.9$ (Region II, III and IV) compared to no extension of the profile (Figure 4.7).

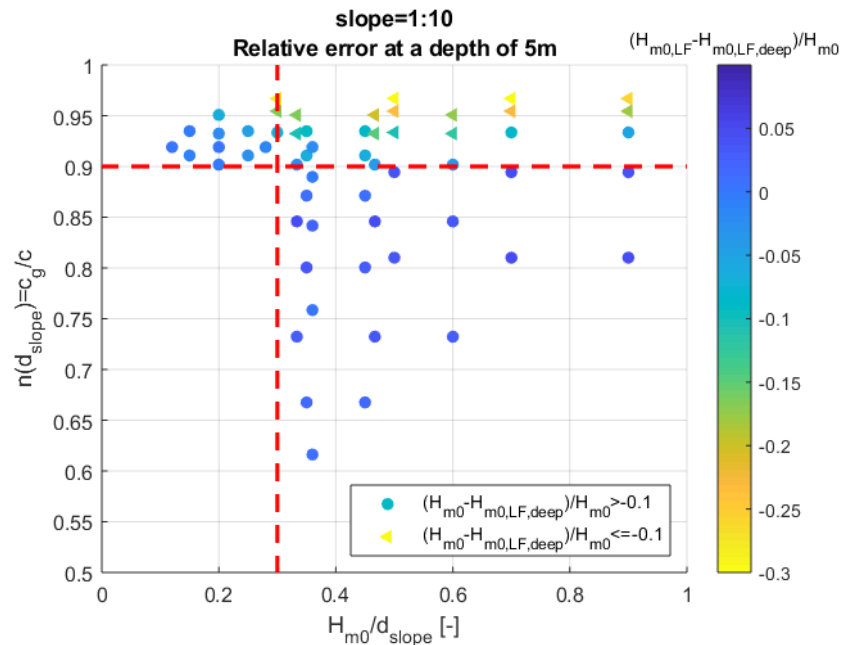


Figure 4.5: Relative error in infragravity wave height at a depth of 5 m as a function of the dimensional wave height and the celerity ratio for an artificial shoreface slope of 1:10. The upper starting depth of the artificial slope (d_{slope}) is used in the dimensional wave height and the celerity ratio. The colours of the markers represent the relative error in infragravity wave height and the symbol markers whether the error is larger than -0.1 (circle) or less than/equal -0.1 (triangle). The red dashed lines show the criteria for which the boundary conditions are not valid.

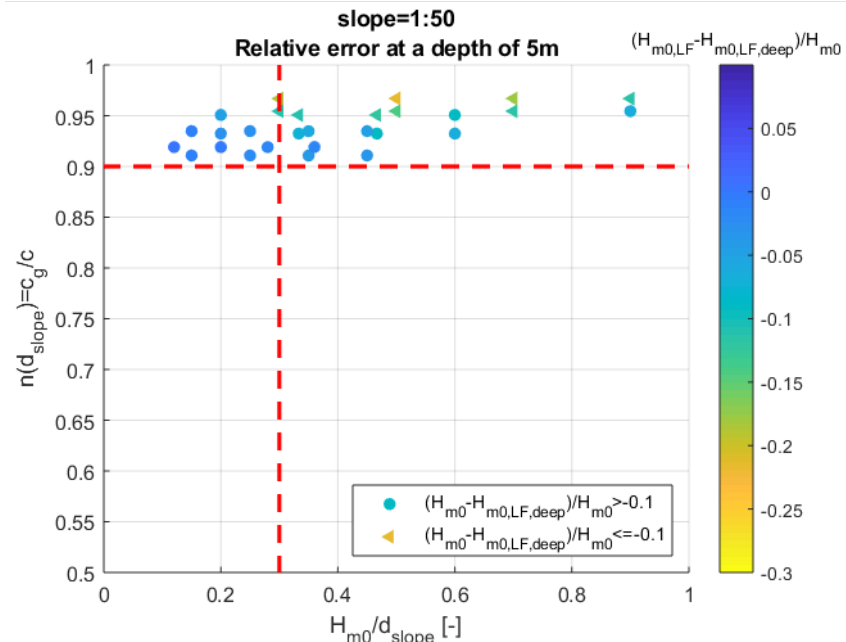


Figure 4.6: Relative error in infragravity wave height at a depth of 5 m as a function of the dimensional wave height and the celerity ratio for a slope of 1:50. The upper starting depth of the artificial slope (d_{slope}) is used in the dimensional wave height and the celerity ratio. The colours of the markers represent the relative error in infragravity wave height and the symbol markers whether the error is larger than -0.1(circle) or less than/equal -0.1 (triangle). The red dashed lines show the criteria for which the boundary conditions are not valid.

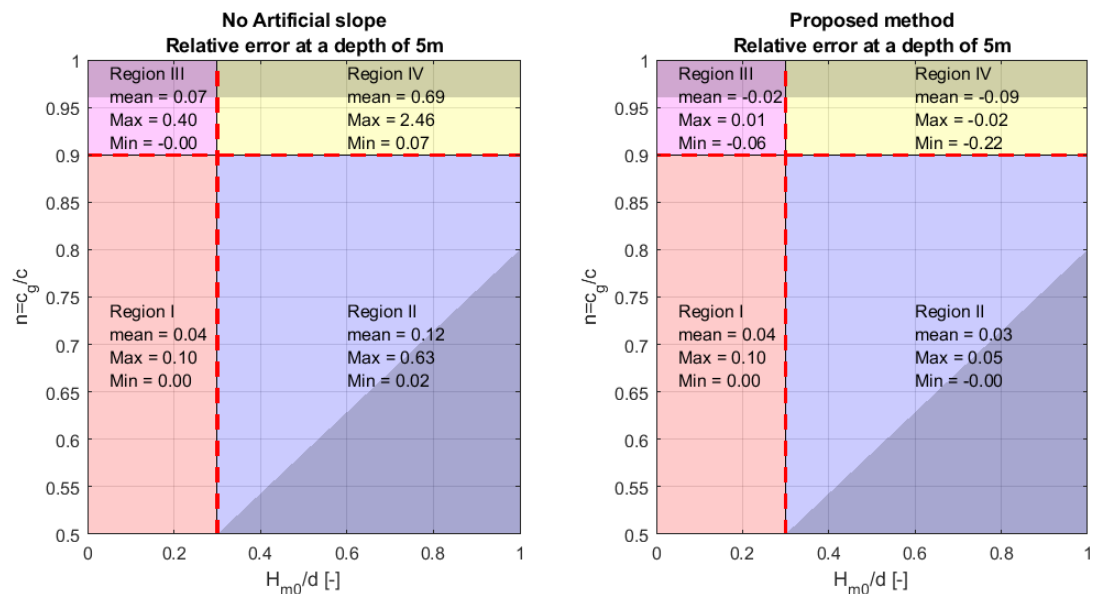


Figure 4.7: Error measures for each of the different regions in the n versus H_{m0}/d plot. The left panel shows the errors when no artificial slopes are applied (d represents the seaward depth of the observed profile). The right panel shows the error when an artificial slope is applied according to the proposed method with the extension with an artificial slope (d represents the upper starting depth of the artificial slope). The regions for which no simulations have been performed are marked in grey. The artificial slopes applied in the proposed method on the right is a 1:10 slope for regions I, II and III and a 1:50 slope for region IV.

4.4 Conclusions

The bound infragravity-wave height prediction by *Hasselmann (1962)* is overestimated in shallow water. When imposing this too large bound infragravity-wave height at the XBeach model boundary, infragravity wave heights in the domain are overestimated as well, leading to overestimations in dune erosion predictions. To correct for this, reduction factor approaches have been identified and examined in Phase II of the current report to be able to prescribe correct infragravity-wave heights at the offshore model boundary. Unfortunately, the reduction of the bound infragravity-wave height on the offshore model boundary is not found to provide a solution for the overestimation of the infragravity-wave height in the nearshore. When a reduction on the infragravity-wave height is applied at the boundary, the infragravity wave subsequently rapidly increases in height in the domain itself because the short-wave forcing is not yet in balance with the local water depth, thereby forcing the strong growth of the infragravity waves. This behaviour is independent of the amount of reduction applied on the infragravity-wave boundary, and thereby all four approaches investigated here will be inapplicable as they only provide a reduction on the infragravity-wave height itself and no information on the correct short-wave groupiness and forcing.

A solution to reduce the number of grid cells and corresponding long calculation times in case of large required offshore water depths at the boundary is found in the implementation of a steeper artificial shoreface slope down to the required water depth. However, as the infragravity-wave growth is sensitive to bed slope, especially in shallow water, certain thresholds have to be respected. A comparison of different wave conditions shows that an artificial slope of 1/10 can be applied when the celerity ratio n at the shallow start-depth (top) of the artificial slope is less than 0.9 or when the dimensional wave height, H_{m0}/d , is smaller than 0.3. When the wave celerity ratio n at the starting depth of the artificial slope > 0.9 (typically long short-wave periods) and $H_{m0}/d > 0.3$, a gentler 1/50 slope is recommended. Following this approach results in maximum relative errors in nearshore infragravity-wave height of 20% with most of the relative errors smaller than 10%. It is important to note that conditions with $n > 0.96$ have not been tested, but that this would be for cases with very shallow start water depths where a specific approach '*Toets op Maat*' is required. Additionally, as the given offshore wave conditions are generally defined at 20 m depth it is important to adapt the short-wave height applied at the offshore boundary, when starting in shallower or deeper water, following de-shoaling.

5 Conclusions and recommendations

5.1 Overall findings

The main research question of the study was: *“How should wave boundary conditions be applied in the Dutch dune safety assessment to ensure a robust and accurate description of infragravity waves in a time-efficient manner?”*

To avoid overestimation of the infragravity-wave height in the nearshore zone and ensure robust results, in this study, the required boundary condition depth has been determined at about three times the offshore short-wave height when respecting the wave celerity ratio $n < 0.9$ ($n = c_g / c$). In addition, to promote a time-efficient approach, we identified that in the absence of local available profile data, the number of computational grid cells and corresponding calculation times may be reduced considerably by introducing a steep artificial slope of 1/10 where the wave celerity ratio $n < 0.9$ and $H_{m0}/d > 0.3$ or $n > 0.9$ and $H_{m0}/d < 0.3$, and a more gentle 1/50 slope for $n > 0.9$ and $H_{m0}/d > 0.3$, down to the required offshore boundary depth. The application of a reduction factor on the bound infragravity-wave height on the offshore model boundary was also investigated but was seen to not provide a solution for the overestimation of the infragravity-wave height in the nearshore when starting in shallower water than recommended.

The two sub questions are answered in more detail here below.

5.1.1 Research question 1

“At what minimum water depth should the XBeach boundary be placed to avoid overestimation of the infragravity-wave height?”

The required start depth is about three times the offshore short-wave height ($H_{m0}/d < 0.3$) when respecting the wave celerity ratio $n < 0.9$ ($n = c_g / c$). In general, when respecting < 10% infragravity-wave height overestimation, resulting dune-erosion volume overestimations are generally < 10%. Including directional spread demonstrates that infragravity-wave heights and resulting dune erosion volumes are considerably less overestimated (in the order of only a few %) when starting XBeach simulations in shallow water, compared to profile-mode.

5.1.2 Research question 2

“How could the infragravity-wave boundary condition be modified to allow for application starting closer to shore?”

To promote a time-efficient approach, we identified that the number of computational grid cells and corresponding calculation times can be reduced considerably by introducing an artificial slope of 1/10 down to the required offshore boundary depth when the wave celerity ratio $n < 0.9$ and $H_{m0}/d > 0.3$ or $n > 0.9$ and $H_{m0}/d < 0.3$, and a 1/50 slope for $n > 0.9$ and $H_{m0}/d > 0.3$. The application of a steeper artificial slope than observed is only recommended in the absence of local available profile data down to the required start-depth, as is for instance the case at the Wadden Coast or in Zeeland. Especially at those locations, the 20 m depth contour is present relatively far from the shoreline and profile data is often not available. The application of a steep shoreface slope at those sites will permit a considerable reduction in grid length and in corresponding calculation time.

The application of a reduction factor on the bound infragravity-wave height on the offshore model boundary was also investigated but was seen to not provide a solution for the overestimation of the infragravity-wave height in the nearshore when starting in shallower water than recommended.

When applying a reduction factor on the infragravity-wave height at the offshore boundary, the infragravity wave subsequently increases rapidly in height in the domain itself. This is because the short-wave forcing is not yet in balance with the local water depth, which induces - through the too large wave-group forcing - a too large infragravity-wave.

5.2 Recommendations for BOI

5.2.1 Implications for model setup

- The required start depth is at least about three times the offshore short-wave height ($H_{m0}/d < 0.3$) and should respect the wave celerity ratio $n < 0.9$ ($n = c_g/c$).
- For locations where the available cross-shore profiles do not extend up to the required offshore water depth, it is recommended to apply a steep shoreface slope down to the required water depth. A 1/10 slope is acceptable if $n < 0.9$ and $H_{m0}/d > 0.3$ or $n > 0.9$ and $H_{m0}/d < 0.3$, and a 1/50 slope for $n > 0.9$ and $H_{m0}/d > 0.3$, where the water depth (d) is defined at the seaward end of the observed cross-shore profile.
- Apply (de-)shoaling from the 20 m water depth contour to the recommended water depth, using the representative wave period in XBeach ($T_{m-1,0}$) to compute the shoaling coefficient.

The procedure to check whether a domain extension is required and how this extension should be applied is shown in Figure 5.1. This procedure results in three adaptation approaches for different regions in the n versus H_{m0}/d plot (see Figure 5.2). Based on the offshore water depth (including surge) of the profile, it can be verified whether the profile needs to be extended. When the profile requires an extension, this profile is extended to a start depth (d_{start}) defined by the deeper of $H_{m0}/d=0.3$ and $n=0.9$. This water depth is given by,

$$d_{start} = \max[3.3 H_{m0,shoal}; n(d) < 0.9] \quad (5.1)$$

Where $H_{m0,shoal}$ is the shoaled offshore wave height and $n^{-1}(d)$ is the water depth for which the celerity ratio equals 0.9. Furthermore, the offshore wave height needs to be de-shoaled to the start depth from the water depth at which the wave conditions are provided (typically the 20 m depth contour for the Dutch dune safety assessment). With respect to a water depth of 20m, the de-shoaled wave height is given by,

$$H_{m0,shoal} = \sqrt{\frac{c_g(d=20\text{ m})}{c_g(d_{start})}} H_{m0} \quad (5.2)$$

Where $H_{m0,shoal}$ is the de-shoaled wave height, c_g the group velocity at a given depth and H_{m0} the offshore wave height.

5.2.2 Recommendations for further development and research in next steps of the Action Plan

- The cross-shore grid resolution should be evaluated for shorter cross-shore grid lengths (see Appendix A.1).
- The inclusion of directional spread in a 1D simulation (through a parameterized reduction of the resulting infragravity-wave height) should be further examined in subproject "Approaches to reduce calculation time".
- A close evaluation of the parameter $epsi$ is recommended. This parameter controls the forcing of the mean (tidal) current. In the current study was observed that this parameter can be the source of erroneous additional low-frequency energy. It is therefore currently set to 0 to prevent additional artificial long wave generation.
- The linear model of *Reniers et al. (2002)* is showing potential, as it can help to efficiently propagate the short-waves in to shallower water, and thereby change the local forcing in the model. However, presently the model is not yet providing the required detailed information on short-wave groups needed to force the infragravity wave. In future dune safety assessments

beyond 2023, the linear model may have developed further and could perhaps be implemented to provide new short-wave and long-wave boundary conditions for the XBeach model, including short-wave groupiness.

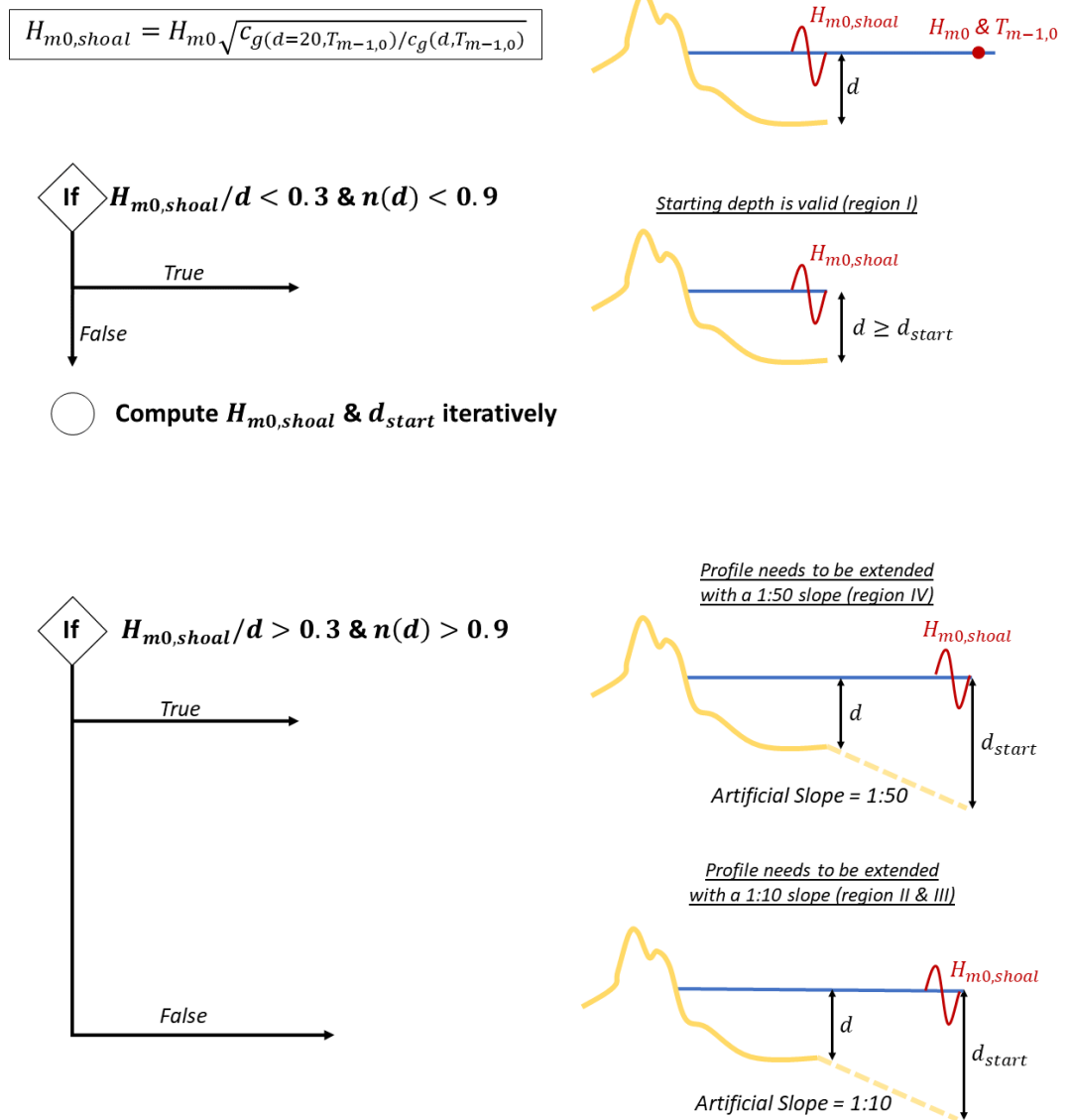


Figure 5.1: Overview of the procedure to determine whether the profile needs to be extended. The regions mentioned in the figure correspond to the four regions in the n versus H_{m0}/d plot (see Figure 5.2). The wave period ($T_{m-1,0}$) and wave height (H_{m0}) are obtained from the hydraulic conditions at a depth of 20 m. The wave celerity and group velocity are computed with the formulas from linear wave theory. The iterative method which is used to solve the starting depth and shoaled wave height is presented in Appendix A.2.

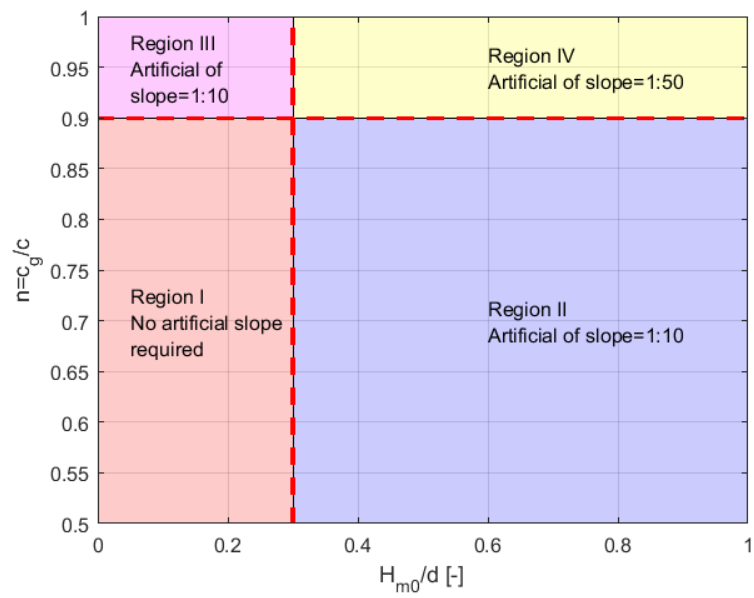


Figure 5.2: Adaptation approach for the different regions in the n versus H_{m0}/d plot.

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A.1 Grid resolution

The cross-shore grid resolution is observed to be an important parameter in the evolution of short-wave groupiness. A too coarse grid might lead to numerical damping in large spatial domains, as shown in Figure A.1-1 (note the total cross-shore extent of 35 km).

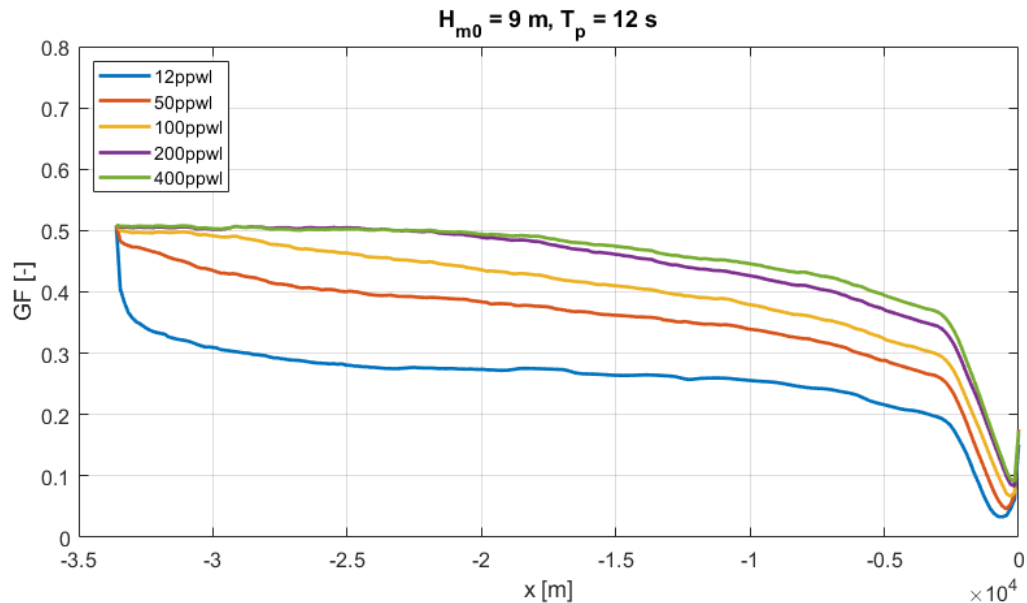


Figure A.1-1 Groupiness factor through the domain for different grid resolutions, varying from 12 to 400 points per wave length for storm wave condition of $H_{m0} = 9$ m and $T_p = 12$ s.

As the short-wave groupiness is forcing the infragravity waves, the damping of the short-wave groupiness is translated in a substantial (~20%) difference of the infragravity-wave height in the nearshore for a large cross-shore domain (Figure A.1-2). Therefore, especially (or perhaps only) for long computational grids it is strongly recommended to investigate the importance of numerical damping on dune erosion. This is especially important for the shorter ($T_p < 12$ s) offshore wave periods which are found to be more sensitive to the grid resolution than larger period waves. Further investigation is recommended in the following BOI project steps to see how the grid set-up could be optimized for domains typical for the Dutch dune safety assessment. Currently, the cross-shore grid size resolution for XBeach grid setup is determined using the Matlab function from the OpenEarthTools toolbox (revision 14057).

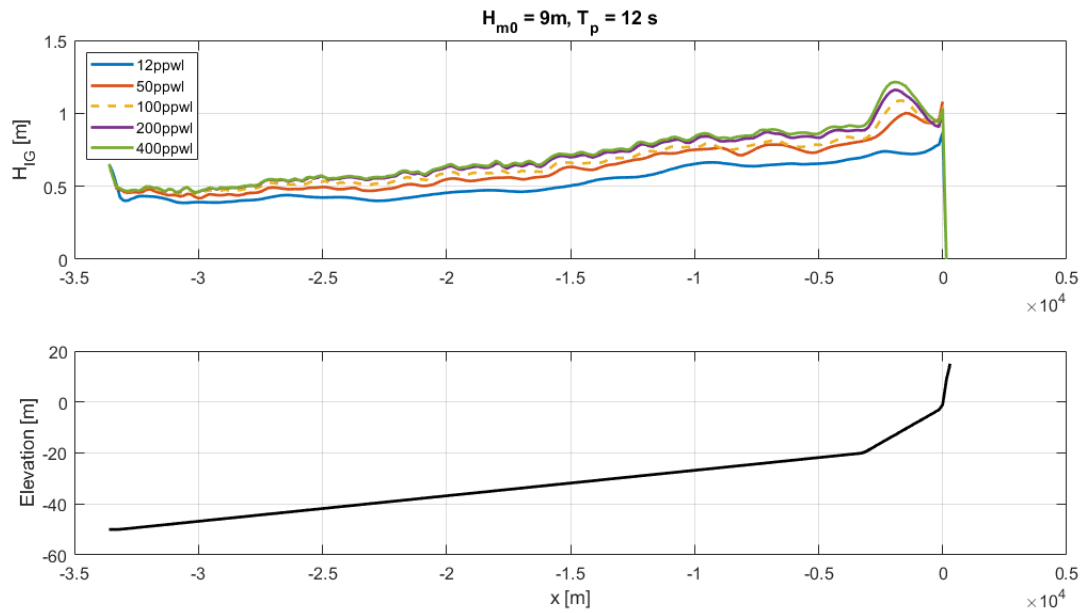


Figure A.1-2 Infragravity-wave height predictions through the domain for different grid resolutions, varying from 12 to 400 points per wave length for storm wave condition of $H_{m0} = 9\text{ m}$ and $T_p = 12\text{ s}$.

A.2 Iterative method

The starting depth cannot directly be computed but needs to be solved iteratively. The recommended approach is described in Figure A.2-1.

First, the depth for which the wave celerity ratio (n) equals 0.9 is computed by increasing a dummy depth until the wave celerity is equal or smaller than 0.9. The next step is to compute the starting depth iteratively based on the predefined requirements ($H_{m0}/d < 0.3$ and $n < 0.9$). Since the starting depth is related to shoaled wave height, an initial shoaled wave height is required. This initial estimate is equal to the shoaled wave height at the depth of the seaward end of the observed profile.

In the next iteration, the shoaled wave height at the starting depth of the previous iteration is used. The spectral period is applied to compute the shoaled wave height because XBeach computes the energy transport for the spectral period. The wave celerity ratio (n) is computed based on the peak period.

$$\begin{aligned}d_{start} &= d \\d_{start,previous} &= 2d \\d_{dummy} &= d_{start} \\ \text{While } n(T_p, d_{dummy}) &> 0.9 \\ & \quad d_{dummy} = d_{dummy} + 0.05 \\ & \quad d_{n=0.9} = d_{dummy} \\ \text{While } \text{abs}(d_{start} - d_{start,previous}) &> 0.05 \\ & \quad d_{start,previous} = d_{start} \\ & \quad d_{start} = \max(3.33H_{m0,shoal}; d_{n=0.9}) \\ & \quad H_{m0,shoal} = H_{m0} \sqrt{c_g(d = 20, T_{m-1,0}) / c_g(d_{start}, T_{m-1,0})}\end{aligned}$$

Figure A.2-1: Iterative method to solve for the starting depth and shoaled wave height. This method requires the water depth at the seaward end of the observed profile (d), the shoaled wave height at the seaward end of the observed profile ($H_{m0,shoal}$), the peak period (T_p) and the spectral period ($T_{m,10}$), as input